

الف) $y = x^2 - 5 \Rightarrow x^2 = y + 5 \Rightarrow x = \pm \sqrt{y + 5} \Rightarrow R_f = [-5, +\infty)$

ب) $y = x^3 + 1 \Rightarrow x^3 = y - 1 \Rightarrow x = \sqrt[3]{y - 1} \Rightarrow R_f = \mathbb{R}$

الف) $y = x^2 - 2x + 4$

$y = (x^2 - 2x + 1) + 3 \Rightarrow y - 3 = (x - 1)^2 \Rightarrow x - 1 = \pm \sqrt{y - 3} \Rightarrow R_f = [3, +\infty)$

ب) $y = x^2 - 5x + 1$

$x^2 - 5x + 1 - y = 0 \Rightarrow x = \frac{5 \pm \sqrt{25 - 4(1 - y)}}{2} = \frac{5 \pm \sqrt{21 - 4(1 - y)}}{2}$
 $25 - 4(1 - y) \geq 0 \Rightarrow 21 - 4(1 - y) \geq 0 \Rightarrow 21 - 4 + 4y \geq 0 \Rightarrow 17 + 4y \geq 0 \Rightarrow y \geq -\frac{17}{4}$
 $R_f = \left[-\frac{17}{4}, +\infty\right)$

الف) $y = \frac{x^2 + 2}{x^2 - 1} \Rightarrow y(x^2 - 1) = x^2 + 2 \Rightarrow yx^2 - y = x^2 + 2 \Rightarrow yx^2 - x^2 = y + 2 \Rightarrow x^2(y - 1) = y + 2 \Rightarrow x^2 = \frac{y + 2}{y - 1}$
 $R_f = \left(-\infty, -\frac{1}{2}\right] \cup \left(\frac{1}{2}, +\infty\right)$

ب) $y = \frac{2|x| + 1}{|x| - 2}$
 $y(|x| - 2) = 2|x| + 1 \Rightarrow y|x| - 2y = 2|x| + 1 \Rightarrow y|x| - 2|x| = 2y + 1 \Rightarrow |x|(y - 2) = 2y + 1 \Rightarrow |x| = \frac{2y + 1}{y - 2}$
 $R_f = \left(-\infty, -\frac{1}{2}\right] \cup \left(\frac{1}{2}, +\infty\right)$

$y = \frac{1}{x^2 - 4x} \Rightarrow y(x^2 - 4x) = 1 \Rightarrow yx^2 - 4yx - 1 = 0$

$14y^2 + 4y \geq 0 \Rightarrow 7y^2 + 2y \geq 0 \Rightarrow y(7y + 2) \geq 0 \Rightarrow y \geq 0$

$7y(7y + 2) \geq 0 \Rightarrow y \geq 0$
 $R_f = \left(-\infty, -\frac{1}{2}\right] \cup \left(\frac{1}{2}, +\infty\right)$

الف) $y = x^2 - 4x + 2 \Rightarrow y = (x - 2)^2 - 2 \Rightarrow y + 2 = (x - 2)^2 \Rightarrow x - 2 = \pm \sqrt{y + 2} \Rightarrow x = 2 \pm \sqrt{y + 2}$
 $R_f = [-2, +\infty)$

ب) $y = -x^2 + 4x + 2 \Rightarrow y = -(x - 2)^2 + 6 \Rightarrow y - 6 = -(x - 2)^2 \Rightarrow (x - 2)^2 = 6 - y \Rightarrow x - 2 = \pm \sqrt{6 - y} \Rightarrow x = 2 \pm \sqrt{6 - y}$
 $R_f = (-\infty, 6]$

الف) $y = \sqrt{x^2 - 4x + 4} \rightarrow \begin{cases} x_{\min} = -\frac{b}{2a} = 2 \\ y_{\min} = -1 \end{cases} \quad R_f = \sqrt{[0, +\infty)} = [0, +\infty)$

ب) $y = \sqrt{-x^2 + 4x + 4} \rightarrow \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{\max} = 4 \end{cases} \quad R_f = \sqrt{(-\infty, 4]} = [0, \sqrt{4}]$

الف) $y = x^4 + 3x^2 + 2x + 1 \quad R_f = \mathbb{R}$

ب) $y = \sqrt{x^4 + 3x^2 + 4x + 1} \rightarrow R_f = [0, +\infty)$

الف) $y = \frac{3x+1}{x^2} \quad R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} = \mathbb{R} - \{3\} = R_f$

ب) $y = \sqrt{\frac{3x+1}{x+3}} \rightarrow R_f = \sqrt{\mathbb{R} - \{3\}} \Rightarrow R_f = [0, +\infty) - \{3\}$

الف) $y = \frac{3x+1}{x-2} \quad \begin{matrix} \text{رشته منقسم: میانبندگی} \\ \text{میانبندگی} = \frac{a}{c} \end{matrix} \Rightarrow \begin{matrix} 3 \\ -\frac{1}{2} \end{matrix}$

ب) $y = \frac{2x-2}{1-2x} \Rightarrow y = \frac{-2(1-2x)}{1-2x} = -2 \quad R_f = \{-2\}$

الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \Rightarrow R_f = [2, +\infty)$

ب) $y = \sqrt{\frac{x^2+1}{x}} \Rightarrow y = \sqrt{x + \frac{1}{x}} \quad R_f = \sqrt{[2, +\infty) \cup [1, +\infty)}$

$\frac{x^2+1}{x} = \frac{x^2}{x} + \frac{1}{x}$