

الف)  $y = n^2 - \delta \rightarrow y + \delta = n^2 \rightarrow \sqrt{y + \delta} = n \rightarrow y + \delta \geq 0 \Rightarrow y \geq -\delta$

$R_f = [-\delta, +\infty)$

ب)  $y = n^2 + 1 \rightarrow y - 1 = n^2 \rightarrow \sqrt{y - 1} = n \Rightarrow R_f = [1, +\infty)$

الف)  $y = n^2 - 4n + 2 \rightarrow y = (n - 2)^2 - 2 \Rightarrow y + 2 = (n - 2)^2 \Rightarrow \sqrt{y + 2} = n - 2 \Rightarrow y + 2 \geq 0 \Rightarrow y \geq -2$

$R_f = [-2, +\infty)$

ب)  $y = n^2 - 8n + 1 \rightarrow 0 = n^2 - 8n + (y + 1) \Rightarrow \Delta = b^2 - 4ac = 64 - 4(y + 1) \geq 0 \Rightarrow 16 \geq y + 1 \Rightarrow y \leq 15$

$n = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow \frac{-(-8) \pm \sqrt{64 - 4(y + 1)}}{2} \Rightarrow 4 \pm \sqrt{15 - y} \geq 0 \Rightarrow 4 \geq \sqrt{15 - y} \Rightarrow 16 \geq 15 - y \Rightarrow y \geq -1$

الف)  $n^2 - 9n^2 = -2y - 3 \rightarrow n^2(1 - 9) = -2y - 3 \rightarrow \frac{-2y - 3}{-8} = n^2 \Rightarrow \frac{-2y - 3}{-8} \geq 0 \Rightarrow -2y - 3 \leq 0 \Rightarrow y \geq -\frac{3}{2}$

$\Rightarrow R_f = [-\frac{3}{2}, +\infty)$

ب)  $x|n| + 1 = y|n| - 4y \rightarrow x|n| - y|n| = -4y - 1 \rightarrow |n|(x - y) = -4y - 1$

$|n| = \frac{-4y - 1}{x - y} \Rightarrow \frac{-4y - 1}{x - y} \geq 0 \Rightarrow \frac{-4y - 1}{x - y} \geq 0 \Rightarrow (-\infty, -\frac{1}{4}] \cup (x, +\infty)$

الف)  $y = \frac{1}{n^2 - 9n} \rightarrow y(n^2 - 9n) - 1 = 0 \Rightarrow \frac{-b \pm \sqrt{\Delta}}{2a} \quad \Delta = b^2 - 4ac$

$\frac{9y \pm \sqrt{144y^2 + 4y}}{2a} \Rightarrow 144y^2 + 4y \geq 0 \Rightarrow 4y(36y + 1) \geq 0 \Rightarrow 36y + 1 \geq 0 \Rightarrow y \geq -\frac{1}{36}$

$4y \geq -1 \Rightarrow y \geq -\frac{1}{4} \Rightarrow R_f = [-\frac{1}{4}, +\infty)$

الف)  $y = n^2 - 9n + 2 \rightarrow \frac{-b}{2a} \rightarrow \frac{9}{2} = 4.5 \rightarrow y_{min} = -4.5 \rightarrow R_f = [-4.5, +\infty)$

$R_f = [-4.5, +\infty)$

ب)  $y = -n^2 + 4n + 2 \rightarrow \frac{-b}{2a} \rightarrow \frac{-4}{-2} = 2 \Rightarrow \exp \left\{ \begin{matrix} 2 \\ 4 \end{matrix} \right\} \rightarrow y_{max} = 6$

$R_f = (-\infty, 6]$

$$\sqrt{n^2 - 2n + 1} \rightarrow -\frac{b}{2a} = 1 \Rightarrow \text{ext} \begin{cases} 1 \\ -1 \end{cases} \rightarrow \sqrt{[-1, +\infty)} = [0, +\infty) \quad (\text{الف})$$

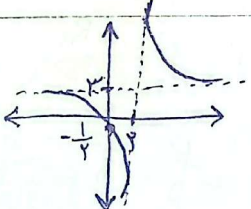
$$\sqrt{-n^2 + 4n + 10} \rightarrow -\frac{b}{2a} = 2 \Rightarrow \text{ext} \begin{cases} 2 \\ -2 \end{cases} \rightarrow \sqrt{(-\infty, 2]} = [0, \sqrt{14}] \quad (\text{ب})$$

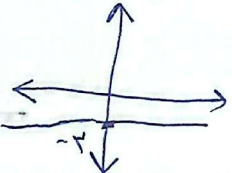
$$y = n^2 + 2n^2 + n + 1 \rightarrow R_f = \mathbb{R} \quad (\text{الف})$$

$$y = \sqrt{n^2 + 2n^2 + n + 1} \rightarrow R_f = [0, +\infty) \quad (\text{ب})$$

$$y = \frac{n+1}{n-2} \rightarrow R_f = \mathbb{R} - \left\{ \frac{2}{1} \right\} \quad (\text{الف})$$

$$y = \sqrt{\frac{n+1}{n+2}} \rightarrow R_f = (-\infty, -1) \cup (-1, +\infty) \quad (\text{ب})$$

$$y = \frac{n+1}{n-2} \rightarrow \left. \begin{matrix} 1 \\ 2 \end{matrix} \right\} \Rightarrow \frac{2}{1} = \frac{a}{c} \quad (\text{الف})$$


$$y = \frac{n-2}{1-2n} \rightarrow y = \frac{2(2n-1)}{-(2n-1)} = -2 \rightarrow y = -2 \quad (\text{ب})$$


$$\cos^2 n + \frac{1}{\cos^2 n} \Rightarrow R_f = [2, +\infty) \quad (\text{الف})$$

$$n + \frac{1}{n} \begin{cases} \text{if } n > 0 \rightarrow n + \frac{1}{n} \geq 2 \quad [2, +\infty) \\ \text{if } n < 0 \rightarrow n + \frac{1}{n} \leq -2 \quad (-\infty, -2] \end{cases} \Rightarrow (-\infty, -2] \cup [2, +\infty) \quad (\text{ب})$$