

الف) $y = u^2 - 8 \Rightarrow u^2 = y + 8 \Rightarrow u = \sqrt{y+8} \quad y+8 \geq 0 \Rightarrow y \geq -8$
 $\Rightarrow R_f = [-8, +\infty)$ ✓

ب) $y = u^2 + 1 \Rightarrow y - 1 = u^2 \Rightarrow \sqrt{y-1} = u \Rightarrow R_f = [1, +\infty)$ ✓

الف) $y = u^2 - 4u + 4 \Rightarrow y = u^2 - 4u + 4 + 2 \Rightarrow y = (u-2)^2 + 2$
 $\Rightarrow y - 2 = (u-2)^2 \Rightarrow \sqrt{y-2} = u-2 \Rightarrow \sqrt{y-2} + 2 = u \quad y-2 \geq 0 \Rightarrow y \geq 2$
 $\Rightarrow R_f = [2, +\infty)$ ✓

ب) $y = u^2 - 8u + 1 \Rightarrow u^2 - 8u + 1 - y = 0$
 $\Delta \geq 0 \Rightarrow 64 - 4(1-y) \geq 0 \Rightarrow 28 \geq 4-y \Rightarrow -21 \leq y$
 $\Rightarrow y \geq -21 \Rightarrow R_f = [-21, +\infty)$ ✓

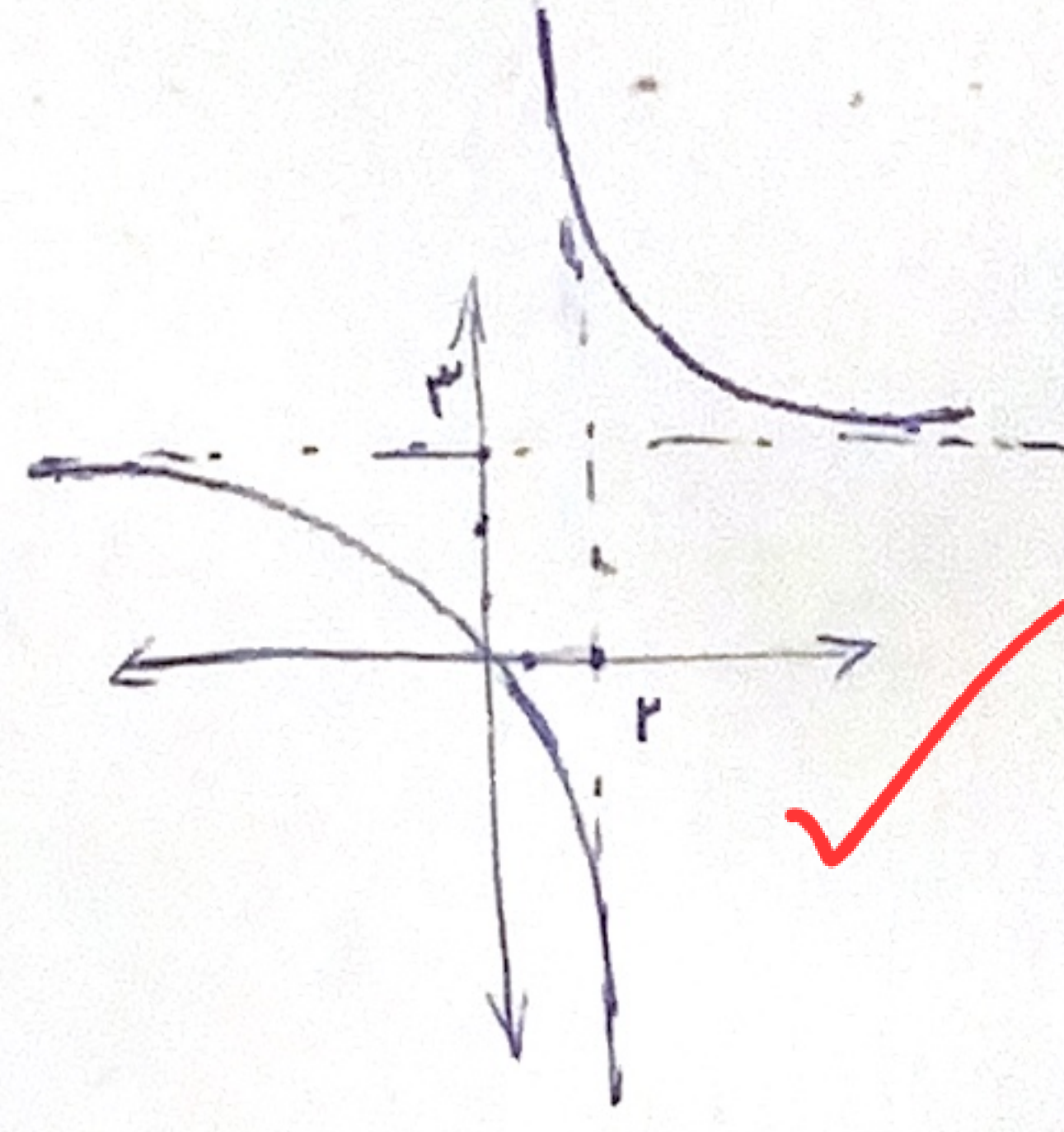
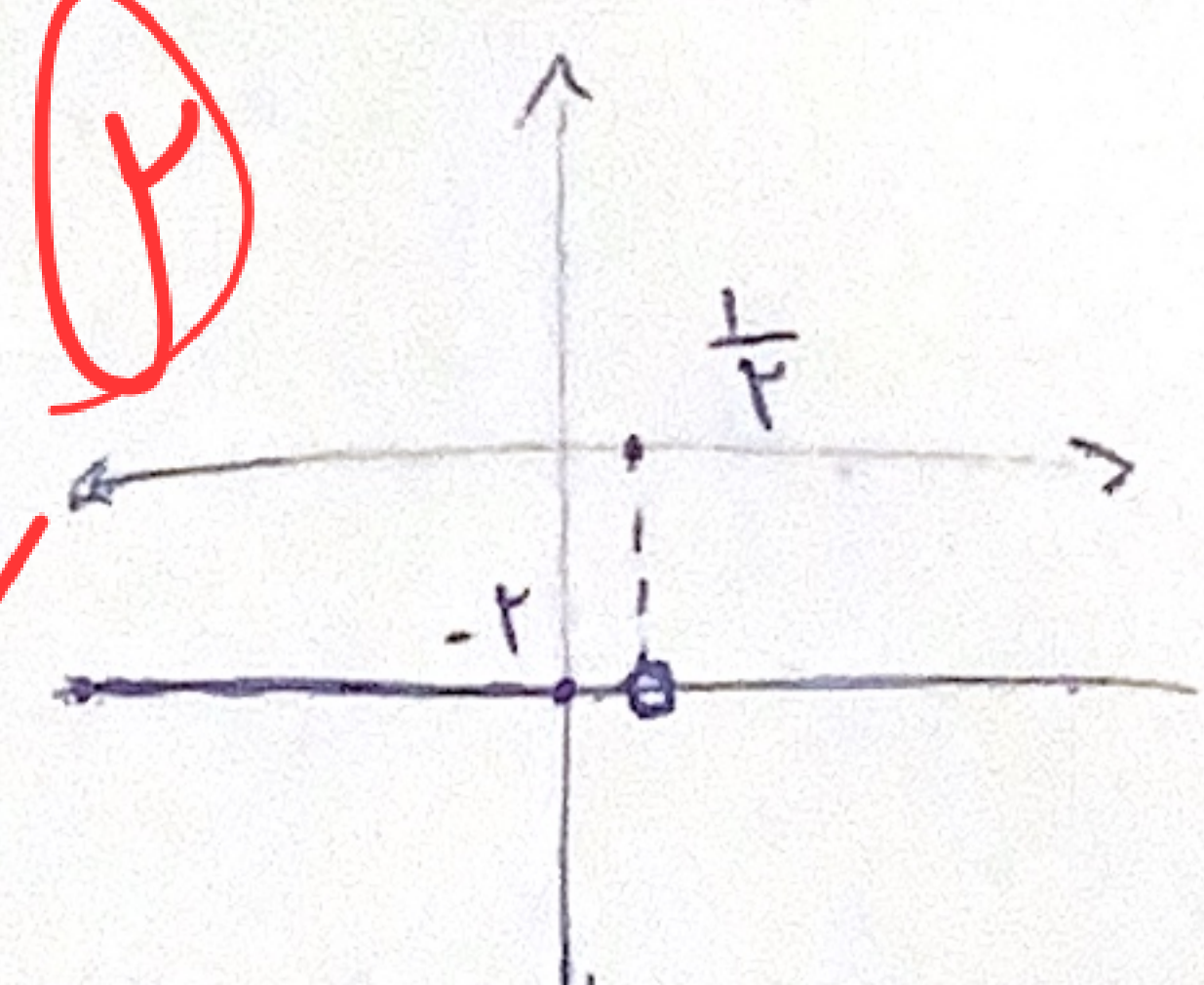
الف) $y = \frac{u^2+2}{u^2-2} \Rightarrow y(u^2-2) = u^2+2 \Rightarrow yu^2 - u^2 = 2y+2$
 $\Rightarrow u^2(y-1) = 2y+2 \Rightarrow u^2 = \frac{2y+2}{y-1} \Rightarrow u = \sqrt{\frac{2y+2}{y-1}}$
 $\Rightarrow \left\{ \begin{array}{l} y-1 > 0 \\ \frac{2y+2}{y-1} \geq 0 \end{array} \right. \Rightarrow R_f = (-\infty, -1) \cup (1, +\infty)$ ✓

ب) $y = \frac{2|u|+1}{|u|-1} \Rightarrow y(|u|-1) = 2|u|+1 \Rightarrow y|u| - 2|u| = 1+y$
 $\Rightarrow |u|(y-2) = 1+y \Rightarrow |u| = \frac{1+y}{y-2} \rightarrow \left\{ \begin{array}{l} y-2 > 0 \\ \frac{1+y}{y-2} \geq 0 \end{array} \right. \Rightarrow R_f = [1, +\infty)$ ✓

الف) $y = \frac{1}{u^2-4u} \Rightarrow 1 = y(u^2-4u) \Rightarrow yu^2 - 4yu - 1 = 0$
 $\Delta \geq 0 \Rightarrow b^2 - 4ac \geq 0 \Rightarrow 16y^2 - 4(-y) \geq 0 \Rightarrow 14y^2 + 4y \geq 0$
 $\Rightarrow 4y(4y+1) \geq 0 \rightarrow \left\{ \begin{array}{l} y \geq 0 \\ y \leq -\frac{1}{4} \end{array} \right. \Rightarrow R_f = (-\frac{1}{4}, 0] \cup [0, +\infty)$ ✓

الف) $y = u^2 - 9u + 2 \xrightarrow{a>0} \begin{cases} u_{min} = \frac{-b}{2a} = \frac{9}{2} \\ y_{min} = -\frac{49}{4} \end{cases} \Rightarrow R_f = [-\frac{49}{4}, +\infty)$ ✓

ب) $y = -u^2 + 6u + 2 \xrightarrow{a<0} \begin{cases} u_{max} = \frac{-b}{2a} = \frac{3}{-1} = -3 \\ y_{max} = 4 \end{cases} \Rightarrow R_f = (-\infty, 4]$ ✓

$y = \sqrt{u^2 - 4u + 1} \quad a > 0 \rightarrow \begin{cases} u_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = -1 \end{cases} \quad a = 1 \wedge + \infty = -1$ $\Rightarrow R_f = \sqrt{[1, +\infty)} = [0, +\infty) \quad \checkmark$ $\text{ب) } y = \sqrt{-u^2 + 4u + 10} \quad a < 0 \rightarrow \begin{cases} u_{\max} = \frac{-b}{2a} = 2 \\ y_{\max} = 14 \end{cases} \quad \textcircled{y} = 2^2 + 4 + 10 = 18$ $\Rightarrow R_f = \sqrt{(-\infty, 14]} = [0, \sqrt{14}] \quad \checkmark$	6
$\text{الف) } y = u^2 + 2u^2 + 2u + 1 \quad \text{بزرگترین توان فرد است} \rightarrow R_f = \mathbb{R} \quad \checkmark$ $\text{ب) } y = \sqrt{u^2 + 4u^2 + 4u + 1} \quad \text{بزرگترین توان فرد است} \rightarrow R_f = \sqrt{\mathbb{R}}$ $\Rightarrow R_f = [0, +\infty) \quad \checkmark \quad \textcircled{y}$	7
$y = \frac{2u+1}{u-2} \quad \text{همواره مثبت است} \rightarrow R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} \Rightarrow R_f = \mathbb{R} - \{2\} \quad \checkmark$ $\text{ب) } y = \sqrt{\frac{2u+1}{u+2}} \Rightarrow R_f = \sqrt{\mathbb{R} - \{2\}} \Rightarrow R_f = [0, +\infty) - \{2\} \quad \checkmark \quad \textcircled{y}$	8
$y = \frac{2u+1}{u-2}$ $\Rightarrow \begin{cases} \text{مجاوب قائم} = 2 \\ \text{مجاوب افقی} = 2 \end{cases}$  $\text{ب) } y = \frac{2u-2}{1-2u} \Rightarrow y = \frac{-2(-2u+1)}{-2u+1}$ $\Rightarrow y = -2 \quad \textcircled{y}$ $1-2u \neq 0 \Rightarrow u \neq \frac{1}{2}$ 	9
$y = \cos^2 u + \frac{1}{\cos^2 u} \Rightarrow R_f = [2, +\infty) \quad \checkmark \quad \textcircled{y}$ $\text{ب) } y = \sqrt{\frac{u^2+1}{u}} = y = \sqrt{u + \frac{1}{u}} \Rightarrow R_f = \sqrt{(-\infty, -1] \cup [1, +\infty)}$ $\Rightarrow R_f = (-\infty, \sqrt{-1}] \cup [\sqrt{1}, +\infty) \quad \checkmark$	10