

الف) $x^2 = y + 5$

$x = \pm \sqrt{y+5}$

$\Rightarrow R_f = [-5, \infty)$

ب) $x^2 = y - 1$

$x = \sqrt{y-1}$

$\Rightarrow R_f = \mathbb{R}$

1

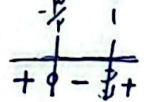
الف) $y = x^2 - 4x + 4 \Rightarrow y = (x-2)^2 \Rightarrow (x-2)^2 = y \Rightarrow x = \sqrt{y} + 2 \Rightarrow R_f = [2, \infty)$

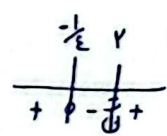
ب) $y = x^2 - 8x + 1 \Rightarrow y = (x^2 - 8x + \frac{16}{4}) - \frac{16}{4} + 1 \Rightarrow y = (x - \frac{8}{4})^2 - \frac{15}{4} \Rightarrow (x - \frac{8}{4})^2 = y + \frac{15}{4}$

$\Rightarrow x = \sqrt{y + \frac{15}{4}} + \frac{8}{4} \Rightarrow R_f = [-\frac{15}{4}, \infty)$

2

الف) $y = \frac{x^2 + 3}{x^2 - 4} \Rightarrow x^2 + 3 = y(x^2 - 4) \Rightarrow x^2 - yx^2 = -3 - 4y \Rightarrow x^2(1-y) = -3 - 4y \Rightarrow x^2 = \frac{-3-4y}{1-y}$

$\Rightarrow x = \pm \sqrt{\frac{-3-4y}{1-y}}$  $R_f = (-\infty, -\frac{3}{4}] \cup (1, \infty)$

ب) $y = \frac{y|x| + 1}{|x| - 4} \Rightarrow y|x| - y|x| = -1 - 4y \Rightarrow |x|(y-4) = -1-4y \Rightarrow |x| = \frac{-1-4y}{y-4}$ 

$\Rightarrow R_f = (-\infty, -\frac{1}{4}] \cup (4, \infty)$

3

$y = \frac{1}{x^2 - 4x} \Rightarrow y(x^2 - 4x) = 1 \Rightarrow x^2y - 4xy - 1 = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{4y \pm \sqrt{16y^2 + 4y}}{2y} \Rightarrow 16y^2 + 4y \geq 0 \Rightarrow 4y(y+4) \geq 0$

$\Rightarrow R_f = [-4, \infty)$

4

الف) $y = x^2 - 4x + 4$

$\text{Min} / x = \frac{-b}{2a} = 2$

$y = 9 - 16 + 4 = -1$

$\Rightarrow \text{Min} / y = -1$

$\Rightarrow R_f = [-1, \infty)$

ب) $-x^2 + 4x + 4$

$\text{Max} / x = \frac{-b}{2a} = 2$

$-4 + 16 + 4 = 16$

$\text{Max} = 16$

$\Rightarrow R_f = (-\infty, 16]$

5

$$\text{الف) } y = \sqrt{x^2 - 2x + 2}$$

$$M_{\min} \left| x = \frac{-b}{2a} = 1 \right. \\ \left. y = -1 \right.$$

$$\Rightarrow R_f = [0, \infty)$$

$$\text{ب) } y = \sqrt{-x^2 + 4x + 1}$$

$$M_{\max} \left| x = \frac{-b}{2a} = 2 \right. \\ \left. y = 1 \right.$$

$$\Rightarrow R_f = [0, 1]$$

4

$$\text{الف) } y = x^4 + 3x^2 + 2x + 1 \\ \Rightarrow R_f = \mathbb{R}$$

$$\text{ب) } y = \sqrt{x^4 + 4x^2 + 4}$$

$$R_f = \sqrt{112} \Rightarrow R_f = [0, \infty)$$

5

$$\text{الف) } y = \frac{3x+1}{x-2}$$

$$R_f = \mathbb{R} - \{3\}$$

$$\text{ب) } y = \sqrt{\frac{4x+1}{2x+3}}$$

$$R_f = \sqrt{112} - \{4\}$$

$$\Rightarrow R_f = [0, \infty) - \{2\}$$

1

9

$$\text{الف) } y = \cos^2 x + \frac{1}{\cos^2 x}$$

$$R_f = [2, \infty)$$

$$\text{ب) } y = \sqrt[3]{\frac{x^2+1}{x}}$$

$$= \sqrt[3]{x + \frac{1}{x}}$$

$$\Rightarrow R_f = (-\infty, -2] \cup [2, \infty)$$

1.