

الف) $x/|x| \geq 0 \Rightarrow x \geq 0 \Rightarrow D_f = [0, +\infty)$ $D_g = \mathbb{R}$ $D_f \neq D_g \Rightarrow$ دو تابع برابر نیستند

ب) $x^2 + \omega x + \nu = 0 \Rightarrow A = b^2 - 4ac = 4\omega - 4\nu = -3 \Rightarrow$ ریشه حقیقی ندارد $\Rightarrow D_f = \mathbb{R}, D_g = \mathbb{R}$ (I)

$$f(x) = \frac{x^2 + 4x + 3 + x + 2}{x^2 + \omega x + \nu} = (I) \Rightarrow f(x) = g(x) \text{ (II) } \xrightarrow{\text{(I), (II)}} \text{دو تابع برابرند}$$

ج) $2 \sin x - 3 = 0 \Rightarrow \sin x = \frac{3}{2}$ $\Rightarrow$ ریشه ندارد $\Rightarrow D_f = \mathbb{R}, D_g = \mathbb{R}$

$$f(x) = \frac{2 \sin x (2 \sin x - 3)}{2 \sin x - 3} \quad (2 \sin x - 3 \neq 0, f(x) = 2 \sin x = g(x)) \Rightarrow \text{دو تابع برابرند}$$

د) $D_f = \mathbb{R} - \{0\}, D_g = \mathbb{R} - \{0\}$ $x^2 = |x| \Rightarrow \frac{x}{|x|} = \frac{|x|}{x} \Rightarrow f(x) = g(x)$ (II)

$\Rightarrow D_f = D_g$ (I) $\xrightarrow{\text{(I), (II)}} f(x) = g(x)$ دو تابع برابرند

الف) $x^2 + 1 \neq 0 \Rightarrow x^2 \neq -1 \Rightarrow D_f = \mathbb{R}, D_g = \mathbb{R}$

$$f(x) = \left[\frac{x^2}{x^2 + 1} \right] = \left[\frac{x^2 + 1 - 1}{x^2 + 1} \right] = \left[1 - \frac{1}{x^2 + 1} \right] = 0$$

$0 \leq x - [x] < 1 \Rightarrow [x - [x]] = 0$ $\Rightarrow$ دو تابع برابرند

ب) if $x = 1/\omega \Rightarrow f(x) = \frac{1}{\omega} \Rightarrow f(x) \neq g(x)$ دو تابع برابر نیستند

ج) $x - |x| > 0 \Rightarrow |x| < x \Rightarrow D_f = \emptyset$

$|x| - x > 0 \Rightarrow |x| > x \Rightarrow D_g = (-\infty, 0)$ $\Rightarrow D_f \neq D_g \Rightarrow$ دو تابع برابر نیستند

د) $\frac{x^2 - x}{x - 1} = \frac{x(x^2 - 1)}{x - 1} = \frac{x(x - 1)(x + 1)}{x - 1} \xrightarrow{x \neq 1} f(x) = x^2 + x = g(x)$

if $x = 1 \Rightarrow g(x) = f(x) = 1$ پس دو تابع با هم برابرند

$\begin{cases} f(x) + g(x) = x^2 + x + 1 \\ f(x) - g(x) = x^2 - x + 1 \end{cases} \Rightarrow 2f(x) = 2x^2 + 2 \Rightarrow f(x) = x^2 + 1$

$g(x) = x$

$f^2(x) - g^2(x) = (x^2 + 1 + x)(x^2 + 1 - x) = (x^2 + 1)^2 - x^2 = x^4 + 1 + 2x^2 - x^2 = x^4 + x^2 + 1$

$f \circ g = (x - \sqrt{x^2 - 4}) \circ (x + \sqrt{x^2 - 4}) = x^2 - (\sqrt{x^2 - 4})^2 = x^2 - x^2 + 4 = 4$

(نسبت دو معادله را از ۲ ضرب کردیم)

$f(x) = g(x) \Rightarrow \frac{ax - b}{2ax^2 - 2ax - 5} = \frac{2am + 4}{2m^2 - 2mx + 2n} \Rightarrow$

$$\begin{cases} 2a = 1 \Rightarrow a = \frac{1}{2} \\ -b = 4 \Rightarrow b = -4 \\ -2m = -3 \Rightarrow m = \frac{3}{2} \\ 2n = -5 \Rightarrow n = -\frac{5}{2} \end{cases}$$

$\Rightarrow am - bn = (\frac{1}{2} \times \frac{3}{2}) - (-4 \times \frac{5}{2}) = \frac{3}{4} - 10 = \frac{3 - 40}{4} = -\frac{37}{4}$

$$f(x)=g(x) \Rightarrow D_f = D_g \quad D_f = \mathbb{R} - \{a\} \quad \left. \begin{array}{l} \Lambda x + b \neq 0 \Rightarrow \Lambda x \neq -b \Rightarrow x \neq -\frac{b}{\Lambda} \Rightarrow D_f = \mathbb{R} - \left\{-\frac{b}{\Lambda}\right\} \end{array} \right\} \Rightarrow a = -\frac{b}{\Lambda}$$

$$\Rightarrow \Lambda a = -b \Rightarrow b = -\Lambda a$$

$$f(x)=g(x) \Rightarrow \frac{bx+r}{\Lambda x+b} = c \Rightarrow \Lambda cm + bc = bx + r \xrightarrow{b=-\Lambda a} \Lambda cm - \Lambda ac = -\Lambda ax + r$$

$$\Rightarrow \Lambda c = -\Lambda a \Rightarrow a = -c, \quad -\Lambda ac = r \Rightarrow \Lambda c^2 = r \Rightarrow c^2 = \frac{r}{\Lambda} \Rightarrow c = \pm \sqrt{\frac{r}{\Lambda}}$$

$$\text{J1, J2 b: } c = \frac{1}{\sqrt{\Lambda}} \Rightarrow a = -\frac{1}{\sqrt{\Lambda}} \Rightarrow b = r \Rightarrow \frac{ab}{c} = \left(-\frac{1}{\sqrt{\Lambda}}\right) \times r \times \sqrt{\Lambda} = (-r) \Rightarrow \frac{ab}{c} = -r$$

$$\text{J3, J4 b: } c = -\frac{1}{\sqrt{\Lambda}} \Rightarrow a = \frac{1}{\sqrt{\Lambda}} \Rightarrow b = -r \Rightarrow \frac{ab}{c} = \left(\frac{1}{\sqrt{\Lambda}}\right) \times (-r) \times (-\sqrt{\Lambda}) = r \Rightarrow \frac{ab}{c} = r$$

$$D_{f+g} = D_f \cap D_g = \{-1, 1, 2\} \Rightarrow f+g = \{(-1, -2), (1, 1), (2, 2)\}$$

$$D_{\frac{fg}{f+g}} = D_{fg} \cap D_{f+g} = \{x | f+g \neq 0\} = \{-1, 1, 2\} = R_{\frac{fg}{f+g}}$$

$$\frac{fg}{f+g} = \{(-1, 2), (1, 1), (2, 2)\} \Rightarrow R_{\frac{fg}{f+g}} = \{2, 1, 2\}$$

$$f=g \Rightarrow a = -1$$

$$ra - rb = 1 \Rightarrow -r - rb = 1 \Rightarrow rb = -r \Rightarrow b = -1$$

$$a - rb = c \Rightarrow c = -1 - r(-1) = 0 \Rightarrow c = 0 \quad d = -1$$

$$d + c = -1 + 0 = -1$$

$$D_f = D_g = \{a\} \Rightarrow \text{مخرج مشترك} \Rightarrow \Lambda \sqrt{b} - x^2 + x - m = 0 \quad \text{مخرج مشترك}$$

$$\Rightarrow 1 - r(-1)(-m) = 0 \Rightarrow rm = 1 \Rightarrow m = \frac{1}{r}$$

$$f(x) = \sqrt{-x^2 + x - \frac{1}{r}} \quad -x^2 + x - \frac{1}{r} = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1}}{-2} = \frac{1}{2} \Rightarrow a = \frac{1}{2}$$

$$\frac{x}{f(x)} = \frac{\frac{1}{2}}{\frac{1}{2}} \Rightarrow b = \sqrt{-\frac{1}{4} + \frac{1}{2} - \frac{1}{4}} = 0 \Rightarrow b = 0 \quad a + b = \frac{1}{2}$$

$$D_g = D_f = \mathbb{R} - \{c\} \Rightarrow f(x) = \frac{rx+a}{(x-c)^2} = \frac{rx+a}{x^2+c^2-2cx} \Rightarrow$$

$$x^2 + c^2 - 2cx = x^2 + 4x + b \Rightarrow -2c = 4 \Rightarrow c = -2$$

$$\frac{r(x+\frac{a}{r})}{x^2+4x+b} = \frac{r}{x^2} \Rightarrow x + \frac{a}{r} = x + 2 \Rightarrow a = 4 \Rightarrow$$

$$(x+2)^2$$

$$a+b+c = 4+9-2 = 11$$