

الف) $x|x| \geq 0 \Rightarrow x \geq 0 \Rightarrow D_f = [0, +\infty)$ $D_g = \mathbb{R}$ $D_f \neq D_g \Rightarrow$ دو تابع برابر نیستند

ب) $x^2 + \omega x + \nu = 0 \Rightarrow A = b^2 - 4ac = 2\omega - 4\nu = -3 \Rightarrow$ ریشه حقیقی ندارد $\Rightarrow D_f = \mathbb{R}, D_g = \mathbb{R}$ $\Rightarrow$ دو تابع برابرند

$f(x) = \frac{x^2 + 4x + 3 + x + 2}{x^2 + \omega x + \nu} = (1) \Rightarrow f(x) = g(x) \quad (II) \quad (III) \Rightarrow$ دو تابع برابرند

ج) $x \sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{x}$ $\Rightarrow$ ریشه ندارد $\Rightarrow D_f = \mathbb{R}, D_g = \mathbb{R}$ $\Rightarrow$ دو تابع برابرند

$f(x) = \frac{x \sin x (x \sin x - 1)}{x \sin x - 1} \quad (x \sin x - 1 \neq 0, f(x) = x \sin x = g(x))$

د) $D_f = \mathbb{R} - \{0\}, D_g = \mathbb{R} - \{0\}$ $x^2 = |x| \Rightarrow \frac{x}{|x|} = \frac{|x|}{x} \Rightarrow f(x) = g(x)$ $\Rightarrow$ دو تابع برابرند

$\Rightarrow D_f = D_g \quad (I) \quad (II) \Rightarrow f(x) = g(x)$

الف) $x^2 + 1 \neq 0 \Rightarrow x^2 \neq -1 \Rightarrow D_f = \mathbb{R}, D_g = \mathbb{R}$ $\Rightarrow$ دو تابع برابر نیستند

$f(x) = \left[\frac{x^2}{x^2 + 1} \right] = \left[\frac{x^2 + 1 - 1}{x^2 + 1} \right] = \left[1 - \frac{1}{x^2 + 1} \right] = 0$

$0 \leq x - [x] < 1 \Rightarrow [x - [x]] = 0$

ب) if $x = 1/\omega \Rightarrow f(x) = \frac{1}{x} \Rightarrow f(x) \neq g(x)$ $\Rightarrow$ دو تابع برابر نیستند

ج) $x - |x| > 0 \Rightarrow |x| < x \Rightarrow D_f = \emptyset$ $\Rightarrow D_f \neq D_g \Rightarrow$ دو تابع برابر نیستند

$|x| - x > 0 \Rightarrow |x| > x \Rightarrow D_g = (-\infty, 0)$

د) $\frac{x^2 - x}{x - 1} = \frac{x(x^2 - 1)}{x - 1} = \frac{x(x - 1)(x + 1)}{x - 1} \xrightarrow{x \neq 1} f(x) = x^2 + x = g(x)$ $\Rightarrow$ دو تابع برابرند

if $x = 1 \Rightarrow g(x) = f(x) = 1$

$\begin{cases} f(x) + g(x) = x^2 + x + 1 \\ f(x) - g(x) = x^2 - x + 1 \end{cases} \Rightarrow 2f(x) = 2x^2 + 2 \Rightarrow f(x) = x^2 + 1$ $\Rightarrow$ $g(x) = x$

$f^2(x) - g^2(x) = (x^2 + 1 + x)(x^2 + 1 - x) = (x^2 + 1)^2 - x^2 = x^4 + 1 + 2x^2 - x^2 = x^4 + x^2 + 1$

$f \circ g = (x - \sqrt{x^2 - 4}) \circ (x + \sqrt{x^2 - 4}) = x^2 - (\sqrt{x^2 - 4})^2 = x^2 - x^2 + 4 = 4$



$f(x) = g(x) \Rightarrow \frac{x - b}{x^2 - 2x - 0} = \frac{2am + 2}{x^2 - 2mx + 2n} \Rightarrow$

$\begin{cases} 2a = 1 \Rightarrow a = \frac{1}{2} \\ -b = 2 \Rightarrow b = -2 \\ -2m = -2 \Rightarrow m = 1 \\ 2n = -0 \Rightarrow n = 0 \end{cases}$

$\Rightarrow am - bn = \left(\frac{1}{2} \times \frac{1}{2}\right) - \left(2 \times \frac{0}{2}\right) = \frac{1}{4} - 0 = \frac{1}{4}$

$$f(x) = g(x) \Rightarrow D_f = D_g$$

$$\lambda x + b \neq 0 \Rightarrow \lambda x \neq -b \Rightarrow x \neq -\frac{b}{\lambda} \Rightarrow D_f = \mathbb{R} - \left\{ -\frac{b}{\lambda} \right\} \Rightarrow a = -\frac{b}{\lambda}$$

$$\Rightarrow \lambda a = -b \Rightarrow b = -\lambda a$$

$$f(x) = g(x) \Rightarrow \frac{bx + r}{\lambda x + b} = c \Rightarrow \lambda cm + b = bx + r \xrightarrow{b = -\lambda a} \lambda cm - \lambda ac = -\lambda ax + r$$

$$\Rightarrow \lambda c = -\lambda a \Rightarrow a = -c, \quad -\lambda ac = r \Rightarrow \lambda c^2 = r \Rightarrow c^2 = \frac{r}{\lambda} \Rightarrow c = \pm \sqrt{\frac{r}{\lambda}}$$

$$\text{J1, J2 b: } c = \frac{1}{\sqrt{\lambda}} \Rightarrow a = -\frac{1}{\sqrt{\lambda}} \Rightarrow b = r \Rightarrow \frac{ab}{c} = \left(-\frac{1}{\sqrt{\lambda}}\right) \times r \times \sqrt{\lambda} = -r \Rightarrow \frac{ab}{c} = -r$$

$$\text{J3, J4 b: } c = -\frac{1}{\sqrt{\lambda}} \Rightarrow a = \frac{1}{\sqrt{\lambda}} \Rightarrow b = -r \Rightarrow \frac{ab}{c} = \left(\frac{1}{\sqrt{\lambda}}\right) \times (-r) \times \sqrt{\lambda} = -r \Rightarrow \frac{ab}{c} = -r$$

$$r \neq -1 \Rightarrow \{(-1, r), (r, r), (r, -1), (0, 0)\} \Rightarrow f(x) = \{(-1, r), (r, r), (r, -1), (0, \frac{1}{\sqrt{\lambda}})\}$$

$$r \neq 1 \Rightarrow \{(r, 1), (r, r), (-1, -1), (0, 0)\} \Rightarrow g(x) = \{(r, r), (r, 1), (-1, -1), (0, 0)\}$$

$$D_{f+g} = D_f \cap D_g = \{-1, r, r\} \Rightarrow f+g = \{(-1, -1), (r, 1), (r, r)\}$$

$$D_{\frac{rg}{r+g}} = D_{rg} \cap D_{f+g} = \{x | f+g = 0\} = \{-1, r, r\} = R_{\frac{rg}{r+g}}$$

$$\frac{rg}{r+g} = \{(-1, r), (r, 1), (r, r)\} \Rightarrow R_{\frac{rg}{r+g}} = \{r, 1, r\}$$

$$f-g \Rightarrow a = -1$$

$$ra - rb = 1 \Rightarrow -r - rb = 1 \Rightarrow rb = -r \Rightarrow b = -1$$

$$a - rb = c \Rightarrow c = -1 - r(-1) = 0 \Rightarrow c = 0$$

$$d + c = -1 + 0 = -1 \Rightarrow d = -1$$

$$D_f = D_g = \{a\} \Rightarrow \text{مخرج مشترك} \Rightarrow \Delta \sqrt{b^2 - 4ac} + a - m = 0 \Rightarrow \text{مخرج مشترك}$$

$$\Rightarrow 1 - r(-1)(-m) = 0 \Rightarrow rm = 1 \Rightarrow m = \frac{1}{r}$$

$$f(x) = \sqrt{-x^2 + x - \frac{1}{r}} \quad -x^2 + x - \frac{1}{r} = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1}}{-2} = \frac{1}{2} \Rightarrow a = \frac{1}{2}$$

$$\frac{a}{f(x)} = \frac{\frac{1}{2}}{-\frac{1}{2}} \Rightarrow b = \sqrt{-\frac{1}{2} + \frac{1}{2} - \frac{1}{2}} = 0 \Rightarrow b = 0 \quad a + b = \frac{1}{2}$$

$$D_g = D_f = \mathbb{R} - \{c\} \Rightarrow f(x) = \frac{rx + a}{(x - c)^2} = \frac{rx + a}{x^2 + c^2 - 2cx} \Rightarrow$$

$$rx + c^2 - 2cx = rx + 4x + b \Rightarrow -2c = 4 \Rightarrow c = -2$$

$$\frac{r(x + \frac{a}{r})}{rx + 4x + 9} = \frac{r}{x + 1} \Rightarrow \frac{r}{x + 1} = \frac{r}{x + 1} \Rightarrow a = 4$$

$$\frac{rx + 4}{x^2 + 4x + 9} = \frac{r}{x + 1} \Rightarrow \frac{r}{x + 1} = \frac{r}{x + 1} \Rightarrow a = 4$$

$$(x + 4)^2$$

$$a + b + c = 4 + 0 - 2 = 2$$