نام و نام خانوادگی آرتین شیروانی فر پاسخنامه تشریحی تکلیف شماره ۸ کلاس ...... B

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الف) $f(x)=\sqrt{x|x|}\ ,\ g(x)=x$

$x|x|\geq 0$
$x\geq 0$

$D_f=[0,+\infty)$
$D_g=\mathbb{R}$ $\longrightarrow D_f\neq D_g$

$f(x)\neq g(x)$

ب) $f(x)=\dfrac{\overbrace{(x+3)(x+1)}^{x^2+4x+3}+x+4}{x^2+5x+7}\ ,\ g(x)=1$

$D_f: \dfrac{x^2+5x+7}{x^2+5x+7}=\{1\}$ $\quad D_g=\{1\}\to D_f=D_g$

$f(x)=g(x)$

ج) $f(x)=\dfrac{4\sin^2x-6\sin x}{2\sin x-3}\ ,\ g(x)=2\sin x$

$D_f=\dfrac{2\sin x\cancel{(2\sin x-3)}}{\cancel{2\sin x-3}}=2\sin x \to D_f=D_g$

$D_g=2\sin x$

$f(x)=g(x)$

د) $f(x)=\dfrac{x}{|x|}\ ,\ g(x)=\dfrac{|x|}{x}$

$D_f=\mathbb{R}-\{0\}$
$D_g=\mathbb{R}-\{0\}$ $\to D_f=D_g\to f(x)=g(x)$

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الف) $f(x)=\left[\dfrac{x^2}{x^2+1}\right]\ ,\ g(x)=[x-[x]]$

$D_f=\mathbb{R}$
$D_g=\mathbb{R}$ $\longrightarrow D_f=D_g$

$f(x)=g(x)$

ب) $f(x)=\dfrac{1}{[2x]}\ ,\ g(x)=\dfrac{1}{2[x]}$

$D_f: 0\leq 2x<1\Rightarrow 0\leq x<\frac{1}{2}$
$D_g: 0\leq x<1$

$D_f\neq D_g$
$f(x)\neq g(x)$

ج) $f(x)=\dfrac{1}{\sqrt{x-|x|}}\ ,\ g(x)=\dfrac{1}{\sqrt{|x|-x}}$

$D_f=\begin{cases}x>0\to 0\times\\ x=0\to 0\times\\ x<0\to \times\end{cases}\ \emptyset$ $\qquad D_g=\begin{cases}x>0\to 0\times\\ x=0\to 0\times\\ x<0\to \checkmark\end{cases}$

$D_g\neq D_f$
$f(x)\neq g(x)$

د) $f(x)=\begin{cases}\dfrac{x^3-x}{x-1}, & x\neq 1\\ 2, & x=1\end{cases}\ ,\ g(x)=x^2+x$

$x^2+x=2$ ✗ $\quad x=1$ ✓

$x^2+x=\dfrac{x^3-x}{x-1}\to x^3-x=x^3-x$ ✓

$D_f=D_g$
$f(x)\neq g(x)$

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$\left.\begin{aligned}f(x)+g(x)&=x^2+x+1\\ f(x)-g(x)&=x^2-x+1\end{aligned}\right\}\to 2f(x)=2x^2+2$

$f(x)=x^2+1\longrightarrow g(x)=x$

$f^2(x)-g^2(x)=(x^2+1)^2-x^2=x^4+2x^2+1-x^2=x^4+x^2+1$

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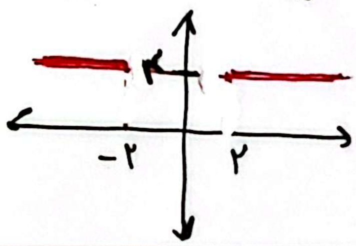
$f\times g=(x-\sqrt{x^2-4})(x+\sqrt{x^2-4})\overset{\text{اتحاد مزدوج}}{\Longrightarrow}=x^2-x^2+4=4$

$D_f=D_g\longrightarrow x^2-4\geq 0\longrightarrow x^2\geq 4$

$\boxed{\begin{aligned}x&\geq 2\\ x&\leq -2\end{aligned}}$



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$g(x)=\dfrac{x-b}{2x^2-3x-5}\longrightarrow 2ax=x\to 2a=1\to a=\frac{1}{2}$

$(g=f)\to -2mx=-3x\to -2m=-3\to m=\frac{3}{2}$

$-b=4\longrightarrow b=-4$

$f(x)=\dfrac{2\times(ax+2)}{2\times(x^2-mx+x)}=\dfrac{2ax+4}{2x^2-2mx+2x}$ $\quad 2x=-5\longrightarrow x=-\frac{5}{2}$

$am-bx=\frac{1}{2}\times\frac{3}{2}-\left(-4\times-\frac{5}{2}\right)=-9.25$

$b=4 \rightarrow \dfrac{4x+2}{8x+4}=\dfrac{1}{2} \rightarrow \dfrac{ab}{c}=\dfrac{-\frac{1}{2}\times 4}{\frac{1}{2}}=-4$

$b=-4 \rightarrow \dfrac{-4x+2}{8x-4}=-\dfrac{1}{2} \rightarrow \dfrac{ab}{c}=\dfrac{\frac{1}{2}\times(-4)}{-\frac{1}{2}}=4$ $\qquad \boxed{c=\pm\dfrac{1}{2}} \qquad \dfrac{ab}{c}=\pm 4$

$g(x)=c$

$f(x)=\dfrac{bx+2}{8x+b}$

$g(x)=f(x) \rightarrow c=\dfrac{bx+2}{8x+b} \rightarrow bx+2=8cx+cb \rightarrow cb=2$ 6

$b=8c \rightarrow c=\dfrac{b}{8} \rightarrow bc=\left(\dfrac{b}{8}\right)b=\dfrac{b^2}{8}=2 \rightarrow b^2=16 \begin{cases} b=4 \\ b=-4 \end{cases}$

$2f-1=\{(-1,3),(2,7),(3,-5),(0,0)\} \rightarrow f=\{(-1,2),(2,4),(3,-2),(0,\frac{1}{2})\}$

$g=\{(2,4),(3,6),(-1,-4),(5,0)\}$

$\dfrac{2g}{f+g}=\dfrac{\{(-1,-8),(2,8),(3,12),(5,0)\}}{\{(-1,-2),(2,8),(3,4)\}}=\{(-1,4),(2,1),(3,3)\}$ 7

$R_{\frac{2g}{f+g}}=\{4,1,3\}$

$f=\{(2,a),(-3,3a-2b),(c,5),(2,d)\} \rightarrow a=d=-1$

$g=\{(2,-1),(-3,1),(a-3b,5)\}$ $\quad 3a-2b=1 \rightarrow -3-2b=1$

$f=g$ $\quad a-3b=c \quad -2b=4$ 8

$c=-1+6=5 \quad b=-2$

$d+c=-1+5=4$

$f(x)=\sqrt{-x^2+x-m} \rightarrow -(x^2-x+m)\geqslant 0 \xrightarrow{\text{به حالت مربع کامل}} -x^2+x-\frac{1}{4}\geqslant 0$

$b=\sqrt{-\frac{1}{4}+\frac{1}{2}-\frac{1}{4}}=\sqrt{0}=0$ 9

$x^2-x+\frac{1}{4}\leqslant 0$ $\quad D_f=\{\frac{1}{2}\}$

$a+b=\frac{1}{2}+0=\frac{1}{2}$ $\quad (x-\frac{1}{2})^2\leqslant 0 \rightarrow x=\frac{1}{2} \rightarrow a=\frac{1}{2}$

$f(x)=\dfrac{2x+a}{x^2+6x+b}=g(x)=\dfrac{2}{x-c} \rightarrow f(x)=\dfrac{2x+a}{(x+\frac{a}{2})(x-c)} \rightarrow D_f=\mathbb{R}-\{c,-\frac{a}{2}\}$

$D_f=D_g \rightarrow D_g=\mathbb{R}-\{c\} \rightarrow c=-\frac{a}{2} \rightarrow (2x+a)(x-c)=2x^2+12x+2b$

$\rightarrow 2x^2+(a-2c)x-ac=2x^2+12x+2b \rightarrow a-2c=12$

$-ac=2b \rightarrow b=\dfrac{-ac}{2}=\dfrac{-2c^2-12c}{2}$ $\quad \hookrightarrow a=12+2c$

$b=-c^2-6c \quad a=-2c=2c+12$

$\rightarrow -4c=12 \rightarrow c=-3$

$\hookrightarrow a+b+c=(2c+12)+(-c^2-6c)+c$

$=6+9-3=12$