

(الف) $m | n | \geq 0 \Rightarrow n \geq 0 \Rightarrow D_f = [0, +\infty), D_g = \mathbb{R} \Rightarrow D_f \neq D_g$ ✓

(ب) $(n+1)(n+1) = n^2 + 2n + 1 \Rightarrow \frac{n^2 + 2n + 1}{n^2 + 2n + 1} = 1 \Rightarrow f(n) = 1 = g(n)$

(ج) $\sin^2 n - \sin^2 n = (\sin n)(\sin n - 1) \Rightarrow \sin n (\sin n - 1) = 0 \Rightarrow \sin n = 0 \Rightarrow f(n) = 0 = g(n)$ ✓

(د) $D_f = \mathbb{R} - \{0\}, D_g = \mathbb{R} - \{0\} \Rightarrow \begin{cases} f(n) = g(n) \text{ if } n > 0 \\ f(n) = g(n) \text{ if } n < 0 \end{cases}$ ✓

(الف) $n^2 + 1 \neq 0 \Rightarrow n^2 \neq -1 \Rightarrow n \neq \pm i \Rightarrow n^2 \geq 0 \Rightarrow \frac{n^2}{n^2 + 1} < 1 \Rightarrow f(n) = 0$ ✓

(ب) $0 \leq n - [n] < 1 \Rightarrow g(n) = 0 \Rightarrow f(n) = 0 = g(n)$ ✓

(ج) $n \neq 0 \Rightarrow n \neq 0 \Rightarrow \begin{cases} [n] = 1 \text{ if } n \in (0, 1) \\ [n] = 0 \text{ if } n \in (1, 2) \end{cases} \Rightarrow g(n) = f(n)$ ✓

(د) $n^2 - n = n(n-1) \Rightarrow \frac{n(n-1)}{n-1} = \frac{n(n-1)(n+1)}{n-1} = n(n+1) = n^2 + n \Rightarrow f(n) = g(n)$ ✓

$f(n) = (n^2 + n + 1) + (n^2 - n + 1) = 2n^2 + 2 \Rightarrow f(n) = n^2 + 1$

$g(n) = (n^2 + n + 1) - (n^2 - n + 1) = 2n \Rightarrow g(n) = n$

$f'(n) - g'(n) = (f(n) - g(n))(f(n) + g(n))$

$f'(n) - g'(n) = (n^2 - n + 1)(n^2 + n + 1)$

$(n^2 - n + 1)(n^2 + n + 1) = n^4 + n^2 + 1 - n^2 = n^4 + 1$

$f(n) = n^2 + 1, g(n) = n \Rightarrow f' - g' = n^4 + 1$ ✓

$(f \circ g) = f(g) = f(n) = (n + \sqrt{n^2 - f}) \cdot (n + \sqrt{n^2 - f}) \Rightarrow$

$f(n) \cdot g(n) = n^2 - (n^2 - f) = f \Rightarrow f \circ g = g \circ f = f$ ✓

$n^2 - mn + n = k(2n^2 - 4n - 8) \Rightarrow k = \frac{1}{4} \Rightarrow -m = -2k = -\frac{1}{2} \Rightarrow m = \frac{1}{2}$

$n = -\delta k = -\frac{\delta}{4} \Rightarrow an + 2 = k(m - b) = \frac{1}{4}(m - b) = \frac{1}{4}n - \frac{b}{4}$

$a = \frac{1}{4}, -\frac{b}{4} = -\frac{\delta}{4} \Rightarrow b = -\delta \Rightarrow am - bn = (\frac{1}{4})(\frac{1}{2}) - (-\delta)(-\frac{\delta}{4}) = \frac{1}{8} - \frac{\delta^2}{4}$ ✓

$$D_f = \{R - \frac{1}{\lambda} b\} \Rightarrow D_f = D_g \Rightarrow -\frac{b}{\lambda} = a \Rightarrow \boxed{\lambda a = -b}$$

$$\lambda a + b \neq 0 \Rightarrow \lambda \neq -\frac{b}{a}$$

$$\frac{b\lambda + r}{\lambda a + b} = c \Rightarrow (\lambda a + b)c = b\lambda + r \Rightarrow \boxed{\lambda ac} + \boxed{bc} = \boxed{b\lambda} + \boxed{r} \quad \begin{matrix} \lambda ac \neq b \\ bc = r \end{matrix}$$

$$\boxed{b = \lambda c} \quad \boxed{b = \frac{r}{c}} \Rightarrow \lambda c = \frac{r}{c} \Rightarrow r = \lambda c^2 \Rightarrow c^2 = \frac{r}{\lambda} = \frac{1}{\lambda} \Rightarrow \boxed{c = \pm \frac{1}{\sqrt{\lambda}}} \quad \boxed{b = \pm \sqrt{r}} \quad \boxed{a = \pm \frac{1}{\sqrt{\lambda}}}$$

$$\boxed{\frac{a}{c} = -\frac{1}{\sqrt{\lambda}} \times \sqrt{r} \times \sqrt{\lambda} = -\sqrt{r}}$$

$$a = d \quad g(r) = -1 = f(r) \quad a = d = -1$$

$$f(-1) - b = 1 \Rightarrow -r - b = 1 \Rightarrow b = -r \rightarrow \boxed{b = -r}$$

$$c = a - r \quad b = -1 - r(-r) = -1 + r = 1 \rightarrow \boxed{c = 1}$$

$$c + d = 1 + (-1) = \boxed{0}$$

$$d + c = r$$

$$D_g = \{R - \frac{1}{\lambda} c\}$$

$$\frac{ra + a}{r^2 + 4a + b} = \frac{r}{n - c} \Rightarrow (ra + a)(n - c) = r^2 n + 4r + b \Rightarrow (-rc + a)n - ac = r^2 n + 4r + b$$

$$\rightarrow -rc + a = r \Rightarrow \boxed{a = r + rc}, -ac = r^2 b \Rightarrow \boxed{b = -\frac{ac}{r^2}} \quad \boxed{c = -r} \quad \boxed{a = r}$$

$$b = -\frac{(r + rc)c}{r^2} \sim b = -rc - c^2 \rightarrow a = c(-r - c) \Rightarrow \boxed{c = r} \quad \boxed{b = 9}$$

$$\boxed{a = 11} \Rightarrow \boxed{a + b + c = 11 + 9 + r = 20} \quad \boxed{a + b + c = 11}$$

$$f(g(n)) \Rightarrow \{(a, b)\} \Rightarrow f(a) = b \quad f(n) = \sqrt{-n^2 + n - m} \quad b = f(a) = \sqrt{-a^2 + a - m}$$

$$-n^2 + n - m \geq 0 \quad -n^2 + n - m = 0 \Rightarrow n^2 - n + m = 0$$

$$\Delta = 0 \Rightarrow -n^2 + n - m = 0 \quad (-1)^2 - f(1)(m) = 0 \Rightarrow 1 - 4m = 0 \Rightarrow m = \frac{1}{4}$$

$$m = \frac{1}{4} \quad n^2 - n + m = 0 \quad n = \frac{1}{2} \quad b = f(a) = \sqrt{-a^2 + a - m} \quad b = \sqrt{-(\frac{1}{2})^2 + \frac{1}{2} - \frac{1}{4}} = \sqrt{0} = 0$$

$$\Rightarrow a = \frac{1}{2} \quad b = 0 \quad a + b = \frac{1}{2}$$

$$r \neq -1 = \{(-1, 2)(1, 4)(3, -6)(0, 0)\} \rightarrow f = \{(-1, 2)(1, 4)(3, -2)(0, 1)\}$$

$$g = \{(1, 4)(3, 4)(-1, -4)(8, 0)\} \rightarrow fg = \{(1, 1)(3, 12)(-1, -1)(8, 0)\}$$

$$\frac{fg}{f+g} = \{(1, 1)(-1, 4)(3, 3)\} \Rightarrow R = \{1, 3, 4\}$$