

الف) $f(x) = \sqrt{x|x|} \rightarrow x|x| \geq 0 \rightarrow \begin{cases} x \geq 0 \\ x = 0 \\ x < 0 \end{cases} \rightarrow D_f = \{0\} \cup \mathbb{R}^+ \rightarrow D_g = \mathbb{R}$

ب) $f(x) = \frac{x^2 + \varepsilon x + \eta + \xi}{x^2 + \delta x + \gamma} = 1, g(x) = 1 \rightarrow$ مساله ۱۰۰

$x^2 + \delta x + \gamma \neq 0 \rightarrow \Delta = \delta^2 - 4\gamma = -12 \rightarrow$ مساله ۱۰۰

ج) $f(x) = \frac{\varepsilon \sin^2 x - \eta \sin x}{\nu \sin x - \omega} \rightarrow \nu \sin x - \omega \rightarrow \sin x \neq \frac{\omega}{\nu} \Rightarrow \sin x \in (-1, 1) \rightarrow D_f = \mathbb{R} \rightarrow D_g = \mathbb{R}$

د) $f(x) = \frac{x}{|x|} \rightarrow x \neq 0, g(x) = \frac{|x|}{x} \rightarrow x \neq 0 \rightarrow D_f = \mathbb{R} - \{0\}, D_g = \mathbb{R} - \{0\}, x > 0 \rightarrow 1, x < 0 \rightarrow -1$

هـ) الف) $f(x) = \left[\frac{x^2}{x^2 + 1} \right] \rightarrow x^2 + 1 \neq 0 \rightarrow x^2 \neq -1 \rightarrow \begin{cases} x = 0 \rightarrow \left[\frac{0}{1} \right] = 0 \\ x \neq 0 \rightarrow \frac{x^2}{x^2 + 1} < 1 \rightarrow \left[\frac{x^2}{x^2 + 1} \right] = 0 \end{cases}$

ب) $f(x) = \frac{1}{\lceil x \rceil} \rightarrow \lceil x \rceil \neq 0 \rightarrow D_f = \mathbb{R} - \{0, \frac{1}{2}\}$
 $g(x) = \frac{1}{\lfloor x \rfloor} \rightarrow \lfloor x \rfloor \neq 0 \rightarrow D_g = \mathbb{R} - \{0, 1\} \rightarrow D_f \neq D_g \rightarrow$ مساله ۱۰۰

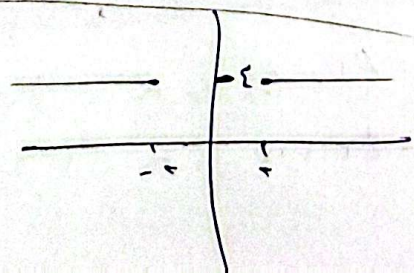
ج) $f(x) = \frac{1}{\sqrt{x} - |x|} \rightarrow x - |x| > 0 \rightarrow x > 0 \rightarrow D_f = \mathbb{R}^+$
 $g(x) = \frac{1}{\sqrt{|x|} - x} \rightarrow |x| - x > 0 \rightarrow D_g = \mathbb{R}^-$

د) $f(x) = \begin{cases} \frac{x^2 - x}{x - 1}; x \neq 1 \\ x; x = 1 \end{cases}, g(x) = x^2 + x \rightarrow g(1) = f(1) = 2 \rightarrow \begin{cases} f(x \neq 1) = \frac{x(x-1)}{x-1} = x \\ g(x \neq 1) = x^2 + x = x(x+1) \end{cases}$

$\begin{cases} f(x) + g(x) = x^2 + x + 1 \\ f(x) - g(x) = x^2 - x + 1 \\ \hline 2g(x) = 2x \rightarrow g(x) = x \\ f(x) + x = x^2 + x + 1 \rightarrow f(x) = x^2 + 1 \end{cases} \rightarrow f^2(x) - g^2(x) = (x^2 + 1)^2 - x^2 = x^4 + 2x^2 + 1 - x^2 = x^4 + x^2 + 1$

$f \circ g = (x - \sqrt{x^2 - \varepsilon})(x + \sqrt{x^2 + \varepsilon}) = x^2 - (x^2 - \varepsilon) = \varepsilon$

$D_f = D_g = (-\infty, -\varepsilon] \cup [\varepsilon, +\infty)$



$$f(x) = \frac{1x + r}{x^r - mx + n} \rightarrow (x+1)(x - \frac{b}{c}) = x^2 - \frac{b}{c}x - \frac{b}{c} \rightarrow m = \frac{b}{c}, n = -\frac{b}{c}$$

$$g(x) = \frac{x-b}{x^r - mx - b} \rightarrow rx^r - mx - b \neq 0 \rightarrow (x-b)(x+1) \neq 0 \rightarrow Dg = \mathbb{R} - \{-1, \frac{b}{c}\}$$

$$\frac{1x + r}{x^r - \frac{b}{c}x - \frac{b}{c}} = \frac{x-b}{x^r - mx - b} \rightarrow rx + r = x - b \rightarrow b = -r$$

$$rx + r = x - b^{x=1} \rightarrow r + r = 1 + b = b \rightarrow a = \frac{1}{c}$$

$$\frac{1x - b}{x^r - \frac{b}{c}x - \frac{b}{c}} = \frac{1}{c} \frac{1x - b}{x^r - \frac{b}{c}x - \frac{b}{c}} - (-\frac{b}{c} \frac{1}{c}) = -\frac{bx}{c}$$

$$\frac{bx + r}{1x + b} = c \rightarrow bx + r = c(1x + b) \begin{cases} c = \frac{1}{c} \\ b = \pm \frac{b}{c} \end{cases}$$

$$bx + r = bc \rightarrow b^r = 1 \Rightarrow b = \pm \frac{b}{c}$$

$$1x + r \neq 0 \rightarrow r(1x + 1) = 0 \rightarrow x \neq -\frac{1}{r} \rightarrow \frac{a}{c} = \frac{(-\frac{1}{c}) \cdot 1}{\frac{1}{c}} = -\frac{1}{c}$$

$$1x - r \neq 0 \rightarrow r(1x - 1) = 0 \rightarrow x \neq \frac{1}{r} \rightarrow \frac{a}{c} = \frac{(-\frac{1}{c}) \cdot (-1)}{\frac{1}{c}} = \frac{1}{c}$$

$$rf = \{(1, 1), (r, 1), (1, -1), (0, 1)\} \rightarrow f = \{(1, r), (r, 1), (1, -r), (0, \frac{1}{c})\}$$

$$\frac{rg}{fg} = \begin{cases} x = -1 \rightarrow \frac{-1}{-1} = 1 \\ x = r \rightarrow \frac{r}{r} = 1 \\ x = \frac{1}{c} \rightarrow \frac{\frac{1}{c}}{\frac{1}{c}} = 1 \end{cases} \rightarrow R = \{1, 1, 1\}$$

$$f = \{(r, a), (-1, 1), (1, b), (r, d)\} \rightarrow a = d$$

$$g = \{(r, -1), (-1, 1), (1, b), (r, d)\}$$

$$a = -1 = d \Rightarrow 1(-1) - rb = 1 \rightarrow b = -\frac{1}{r} \rightarrow d + c = -1 + \frac{1}{c} = \frac{1}{c}$$

$$-1 - r(-\frac{1}{r}) = c \rightarrow c = 0$$

$$-(x^r - x + m) \neq 0 \rightarrow \sqrt{-(x-a)^r} = -x^r + rx - a^r = -x^r + x - m \rightarrow -m^r = -m \rightarrow m = \frac{1}{c}$$

$$ra = 1 \Rightarrow a = \frac{1}{c} \rightarrow b = 0 \rightarrow a + b = \frac{1}{c} + 0 = \frac{1}{c}$$

$$m = 0 \rightarrow \frac{r}{-c} = \frac{m}{b} \rightarrow rb = -rc$$

$$\frac{r}{x-c} = \frac{r(x + \frac{1}{c})}{(x-c)(x + \frac{1}{c})} \rightarrow \frac{r}{-c} = \frac{r}{c} \rightarrow a - rc = 1r$$

$$Df = Dg \Rightarrow R - \{c, -\frac{1}{c}\} = R - \{c\} \rightarrow c = -\frac{1}{c} \rightarrow a = -rc \rightarrow -rc - rc = 1r$$

$$-rc = 1r \rightarrow c = -\frac{1}{c} \rightarrow m = 1, b = 1 \rightarrow a + b + c = 1 + 1 - \frac{1}{c} = 1r$$