محمدرضا امیدی روش

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الف) $f(x)=\sqrt{x|x|}$ ← $x|x|\geq 0$ ← $\begin{cases} x>0 \checkmark \\ x=0 \checkmark \\ x<0 \times \end{cases}$ ← $D_f=\{0\}\cup R^+$ ، $D_g=R$ مساوی نیستند ✓

ب) $f(x)=\frac{x^3+4x^2+3x+x+4}{x^2+5x+7}=1$, $g(x)=1$ ← مساوی هستند ✓

$x^2+5x+7\neq 0$ ← $\Delta=25-28=-3$ ← ریشه ندارد

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ج) $f(x)=\frac{4\sin^2x-6\sin x}{2\sin x-3}$ ← $2\sin x-3$ ← $\sin x\neq \frac{3}{2}$ ← $-1\leq \sin x\leq 1$ ، $D_f=R$ ، $D_g=R$ مساوی هستند ✓

د) $f(x)=\frac{x}{|x|}$ ← $x\neq 0$, $g(x)=\frac{|x|}{x}$ ← $x\neq 0$ ← $D_f=R-\{0\}$ ، $D_g=R-\{0\}$ ، $x>0 \to 1$ ، $x<0\to -1$ مساوی هستند ✓

الف) $f(x)=\left[\frac{x^2}{x^2+1}\right]$ ← $x^2+1\neq 0$ ← $x^2\neq -1$ ← ✓ ← $\begin{cases} x=0 \to [\frac{0}{1}]=0 \\ x\neq 0 \to \frac{x^2}{x^2+1}<1 \to [\frac{x^2}{x^2+1}]=0 \end{cases}$ مساوی هستند ✓

ب) $f(x)=\frac{1}{[2x]}$ ← $[2x]\neq 0$ ← $D_f=R-[0,\frac{1}{2})$

$g(x)=\frac{1}{2[x]}$ ← $[x]\neq 0$ ← $D_g=R-[0,1)$ ← $D_f\neq D_g$ ← مساوی نیست ✓

ج) $f(x)=\frac{1}{\sqrt{x-|x|}}$ ← $x-|x|>0$ ← امکان‌پذیر ندارد ← $D_f=\varnothing$

$g(x)=\frac{1}{\sqrt{|x|-x}}$ ← $|x|-x>0$ ← $D_g=R^-$ مساوی نیست ✓

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د) $f(x)=\begin{cases}\frac{x^3-x}{x-1} ; x\neq 1\\ 2 ; x=1\end{cases}$, $g(x)=x^2+x$ ← $g(1)=f(1)=2$, $f(x\neq 1)=x(x+1)=\frac{x(x-1)(x+1)}{x-1}$

$g(x\neq 1)=x^2+x=x(x+1)$ ✓ مساوی هستند

$$\begin{cases} f(x)+g(x)=x^2+x+1 \\ f(x)-g(x)=x^2-x+1 \end{cases}$$

$2g(x)=2x \to g(x)=x$ ✓

$f(x)+x=x^2+x+1=x^2+1$ ✓

$f^2(x)-g^2(x)=(x^2+1)^2-x^2=x^4+1+x^2$ ✓

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$f\times g=(x-\sqrt{x^2-4})(x+\sqrt{x^2+4})=x^2-(x^2-4)=4$

$D_f=D_g=(-\infty,-2]\cup[2,+\infty)$ ✓

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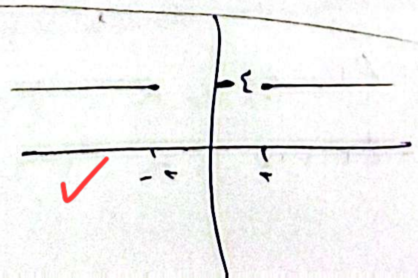

$f(x) = \frac{ax+2}{x^2 - mx + n} \rightarrow (x+1)(x-\frac{5}{2}) = x^2 - \frac{3}{2}x - \frac{5}{2} \rightarrow m = \frac{3}{2}, n = -\frac{5}{2}$

$2x - 5 = 0 \rightarrow x = \frac{5}{2}$

$g(x) = \frac{x-b}{2x^2-3x-5} \rightarrow 2x^2 - 3x - 5 \neq 0 \rightarrow (2x-5)(x+1) \neq 0 \rightarrow D_g = R - \{-1, \frac{5}{2}\}$; $x+1=0, x=-1$

$\frac{ax+2}{x^2-\frac{3}{2}x-\frac{5}{2}} = \frac{x-b}{2x^2-3x-5} \rightarrow 2ax+4 = x-b \xrightarrow{x=0} b = -4$

$2ax+4 = x-b \xrightarrow{x=1} 2a+4 = 1+4 = 5 \rightarrow a = \frac{1}{2}$

$am - bn = \frac{1}{2} \times \frac{3}{2} - (-\frac{5}{2} \times (-4)) = -\frac{37}{4}$

$\frac{bx+2}{ax+b} = c \rightarrow bx+2 = acx+bc \begin{cases} c = \frac{1}{2} \\ b = \pm 4 \end{cases}$

$ad = bc \rightarrow b^2 = 16 = \pm 4$

$ax + 4 \neq 0 \rightarrow 4(2x+1) = 0 \rightarrow x \neq -\frac{1}{2} \rightarrow \frac{ab}{c} = \frac{(-\frac{1}{2}) \times 4}{\frac{1}{2}} = -4$

$ax - 4 \neq 0 \rightarrow 4(2x-1) = 0 \rightarrow x \neq \frac{1}{2} \rightarrow \frac{ab}{c} = \frac{(-\frac{1}{2}) \times (-4)}{\frac{1}{2}} = 4$

$2f = \{(-1,4),(2,8)(3,-4)(0,1)\} \rightarrow f = \{(-1,2)(2,4)(3,-2)(0,\frac{1}{2})\}$

$\frac{4g}{fg} \begin{cases} x=-1 \rightarrow \frac{-8}{-2} = 4 \\ x=2 \rightarrow \frac{8}{8} = 1 \\ x=3 \rightarrow \frac{12}{4} = 3 \end{cases} \rightarrow R = \{4, 1, 3\}$

$f = \{(2,a)(-3, 3a-2b)(c,d)(2,d)\} \rightarrow a = d$

$g = \{(2,-1)(-3,1)(a-3b, 6)\}$

$a = -1 = d \rightarrow 3(-1) - 2b = 1 \rightarrow b = -2$; $-1 - 3(-2) = c \rightarrow c = 5$ $\rightarrow d + c = -1 + 5 = 4$

$-(x^2 - x + m) \geq 0 \rightarrow$ دلتا صفر $\rightarrow \sqrt{-(x-a)^2} = -x^2 + 2ax - a^2 = -x^2 + x - m \rightarrow -a^2 = -m$, $m = \frac{1}{4}$

چون یک جواب دارد

$2a = 1 \rightarrow a = \frac{1}{2} \rightarrow b = 0 \rightarrow a + b = \frac{1}{2} + 0 = \frac{1}{2}$

$x = 0 \rightarrow \frac{2}{-c} = \frac{n}{b} \rightarrow 2b = -nc$

$\frac{2}{x-c} = \frac{2(x+\frac{n}{2})}{(x-c)(x+\frac{a}{2})} \rightarrow \frac{a}{2} - c = 6 \rightarrow a - 2c = 12$

$D_f = D_g \rightarrow R - \{c, -\frac{a}{2}\} = R - \{c\} \rightarrow c = -\frac{a}{2} \rightarrow a = -2c \rightarrow -2c - 2c = 12$

$-4c = 12 \rightarrow c = -3$; $n = 6, b = 9 \rightarrow a + b + c = 6 + 9 - 3 = 12$