

|         |    |    |   |   |   |
|---------|----|----|---|---|---|
|         | -۴ | -۵ | ۰ | ۲ | ۵ |
| $x$     | -  | -  | + | + | + |
| $f(x)$  | +  | ۰  | - | - | ۰ |
| $xf(x)$ | -  | +  | - | - | + |

$\Rightarrow D_f = [-5, 0] \cup [2, 5]$

$$\frac{-3}{-1} + \frac{0}{1} + \frac{2}{2} - \frac{5}{5} = 1$$

$$y = \sqrt{\frac{-x}{f(x)}} \Rightarrow \frac{-x}{f(x)} \Rightarrow f(x) \neq 0$$

برای  $x=0$  و  $x=2$  برابر با صفر

$$D_f = (-3, 0) \cup (0, 2)$$

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if  $x=2 \rightarrow f(2) = 1 \rightarrow f(2) = 1 \rightarrow -f(2) = -1 \rightarrow f(2) = -1$   
 if  $x=-2 \rightarrow f(-2) = 1 \rightarrow f(-2) = 1 \rightarrow f(-2) = 1$

$$f(f(5)) + f(f(1)) = 7 + 2 = 9$$

$$5 - \sqrt{5+4} = 2$$

$$f(x) = \begin{cases} x - \sqrt{x+4} & x > 3 \\ 2x + 3 & x \leq 3 \end{cases}$$

$2 \times 1 + 3 = 5$

$$f(x) = ax^2 - bx + c$$

$$f(x-1) = a(x-1)^2 - b(x-1) + c = ax^2 - 2ax + a - bx + b + c$$

$$f(x-1) - f(x) = ax^2 - 2ax + a - bx + b + c - (ax^2 - bx + c) = -2ax + a + b = 4x$$

$-2a = 4 \Rightarrow a = -2$   
 $-b + a + b = 4 \Rightarrow a = 4$   
 $a - b = -2 - 4 = -6$   
 $b = 4$



$$f(\sqrt{3}-1) = \frac{(\sqrt{3}-1)^2 + 1(\sqrt{3}-1) + 1}{(\sqrt{3}-1)^2 + 1(\sqrt{3}-1) + 1} = \frac{3-2\sqrt{3}+1+\sqrt{3}-1+1}{3-2\sqrt{3}+1+\sqrt{3}-1+1} = \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\boxed{\frac{2-\sqrt{3}}{2-\sqrt{3}} = 1}$$

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$$f\left(x - \frac{1}{x}\right) = \frac{x^2 + 1}{x^2} \Rightarrow \frac{x^2}{x^2} + \frac{1}{x^2} = x^2 + \frac{1}{x^2}$$

$$f\left(x - \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

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$$f\left(x - \frac{1}{x}\right) = x^2 + \frac{1}{x^2} - 2 + 2 \Rightarrow f(x) = x^2 + 2$$

$$f(-3) = (-3)^2 + 2 = 11$$

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$$f(x) = \{x, 0\} \cup \{0, x\}$$

$$g(x) = \sqrt{9-x^2} \rightarrow 9-x^2 \geq 0 \Rightarrow 9 \geq x^2 \Rightarrow \begin{cases} x \geq -3 \\ x \leq 3 \end{cases}$$

$$D(f) = \{(-2, 2) \cup (2, 2)\} \cup \{(2, \sqrt{2}) \cup (2, \sqrt{2})\} \cup \{(0, 3) \cup (0, \sqrt{2})\} \cup \{(1, \sqrt{2})\}$$

$$D(g) = \{(-2, 2) \cup (2, 2)\} \cup \{(2, \sqrt{2}) \cup (2, \sqrt{2})\} \cup \{(0, 3) \cup (0, \sqrt{2})\} \cup \{(1, \sqrt{2})\}$$

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$$a) f(x) = \{(-2, 2) \cup (2, 2)\} \cup \{(2, \sqrt{2}) \cup (2, \sqrt{2})\} \cup \{(0, 3) \cup (0, \sqrt{2})\} \cup \{(1, \sqrt{2})\}$$

$$b) f(x) + 1 = \{(-2, 3) \cup (2, 3)\} \cup \{(2, \sqrt{2}+1) \cup (2, \sqrt{2}+1)\} \cup \{(0, 4) \cup (0, \sqrt{2}+1)\} \cup \{(1, \sqrt{2}+1)\}$$

$$c) 3f(x) + 1 = \{(-2, 7) \cup (2, 7)\} \cup \{(2, 3\sqrt{2}+1) \cup (2, 3\sqrt{2}+1)\} \cup \{(0, 10) \cup (0, 3\sqrt{2}+1)\} \cup \{(1, 3\sqrt{2}+1)\}$$

$$d) f(2x) = \{(-4, 4) \cup (4, 4)\} \cup \{(4, \sqrt{2}) \cup (4, \sqrt{2})\} \cup \{(0, 6) \cup (0, \sqrt{2})\} \cup \{(2, \sqrt{2})\}$$

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$$a) f-g = \{(-2, 2) \cup (2, 2)\} \cup \{(2, \sqrt{2}) \cup (2, \sqrt{2})\} \cup \{(0, 3) \cup (0, \sqrt{2})\} \cup \{(1, \sqrt{2})\}$$

$$b) \frac{f}{g} = \{(-2, 2) \cup (2, 2)\} \cup \{(2, \sqrt{2}) \cup (2, \sqrt{2})\} \cup \{(0, 3) \cup (0, \sqrt{2})\} \cup \{(1, \sqrt{2})\}$$