

$$x \cdot f(x) > 0 \Rightarrow \begin{cases} \text{if } x > 0 \Rightarrow f(x) > 0 \\ \text{if } x < 0 \Rightarrow f(x) < 0 \end{cases} \Rightarrow$$

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$$x > 0 : [0, \infty) \quad \left. \begin{array}{l} f(x) > 0 : [-4, -\infty) \cup [2, \infty) \end{array} \right\} \xrightarrow{+} (x > 0) \times (f(x) > 0) = [2, \infty)$$

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$$x < 0 : [-4, 0] \quad \left. \begin{array}{l} f(x) < 0 : [-\infty, -2] \end{array} \right\} \xrightarrow{-} [-\infty, 0] \Rightarrow (x < 0) \times (f(x) < 0) = [-\infty, -2]$$

$$ID_f = [-\infty, 0] \cup [2, \infty)$$

$$\frac{-x}{f(x)} > 0 \Rightarrow \frac{x}{f(x)} < 0 \Rightarrow \begin{cases} \text{if } x < 0 \Rightarrow f(x) > 0 \\ \text{if } x > 0 \Rightarrow f(x) < 0 \end{cases}$$

$$\hookrightarrow f(x) \neq 0 \Rightarrow x \neq -3 \text{ و } 0, 2 \text{ و } \infty$$

$$ID_f = (-3, 0) \cup (0, 2) \Rightarrow$$

$$-x > 0 \Rightarrow x < 0 : ID_f = [-4, 0]$$

$$x > 0 : ID_f = [0, \infty)$$

$$f(x) > 0 \Rightarrow f(x) > 0 : ID_f = (-3, 0) \cup (2, \infty)$$

$$f(x) < 0 : ID_f = (-\infty, -3) \cup (0, 2)$$

$$f(x) - 2f(2) = x^2 - 2x + 4 \Rightarrow f(2) - 2f(2) = 2^2 - 2 \times 2 + 4 \Rightarrow$$

$$-f(2) = 4 - 4 + 4 = 4 \Rightarrow f(2) = -4$$

$$f(-2) - 2x - 2 = (-2)^2 - 2x - 2 + 4 \Rightarrow f(-2) + 4 = 4 + 2 + 4 \Rightarrow f(-2) = 10$$

$$f(\infty) = \infty - \sqrt{\infty + 4} = \infty - \sqrt{9} = \infty - 3 = \infty$$

$\hookrightarrow \infty > 3$

$$f(2) = 2 \times 2 + 2 \Rightarrow f(2) = 6 \quad \text{و} \quad f(1) = 2 \times 1 + 2 \Rightarrow f(1) = 4 \Rightarrow f(\infty) = 2 \Rightarrow$$

$\hookrightarrow 2 \leq 3$

$\hookrightarrow 1 \leq 2$

$$f(f(\infty)) + f(f(1)) = f(2) + f(4) = 6 + 2 = 8$$

$$f(x-1) = a(x-1)^2 - b(x-1) + 2 \Rightarrow f(x-1) = ax^2 - 2ax + a - bx + b + 2$$

$$f(x-1) - f(x) = (ax^2 - 2ax + a - bx + b + 2) - (ax^2 - bx + 2) \Rightarrow$$

$$(I) f(x-1) - f(x) = ax^2 - 2ax + a - bx + b + 2 - ax^2 + bx - 2 = -2ax + a + b$$

$$(II) f(x-1) - f(x) = 4x + 2 \Rightarrow (II) - (I) \Rightarrow 4x + 2 = -2ax + a + b \Rightarrow 4x = -2ax \Rightarrow$$

$$a - b = -3 - \infty = -\infty \quad \checkmark \quad a + b = 2 \Rightarrow -2 + b = 2 \Rightarrow b = 4 \quad \checkmark \quad -2a = 4 \Rightarrow a = -2$$

$$F(x) = \frac{x^3 + 1x + 5}{x^3 + 4x + 7} = \frac{(x^3 + 4x + 4) + 1}{(x^3 + 4x + 4) + 3} = \frac{(x+1)^3 + 1}{(x+1)^3 + 3} \Rightarrow$$

$$\frac{(\sqrt{3}-1+1)^3 + 1}{(\sqrt{3}-1+1)^3 + 3} = \frac{(\sqrt{3})^3 + 1}{3 + 3} = \frac{3+1}{3+3} = \frac{4}{6} = \frac{2}{3} \quad \checkmark$$

$$\Rightarrow F(\sqrt{3}-1) = \frac{2}{3}$$

$$F(x - \frac{1}{x}) = \frac{x^3}{x^3} + \frac{1}{x^3} = x^3 + \frac{1}{x^3} = (x + \frac{1}{x})^3 \Rightarrow$$

$$(x - \frac{1}{x})^3 = x^3 + \frac{1}{x^3} - 2x \times (x) \times (\frac{1}{x}) = x^3 + \frac{1}{x^3} - 2 \Rightarrow F(x - \frac{1}{x}) - 2 = (x - \frac{1}{x})^3$$

$$x - \frac{1}{x} = -3 \Rightarrow (-3)^3 = F(x - \frac{1}{x}) - 2 \Rightarrow 9 + 2 = F(x - \frac{1}{x}) \Rightarrow F(-3) = 11 \quad \checkmark$$

الف) $\frac{F}{g} = \{(2, \frac{0}{\sqrt{5}}) \text{ و } (1, \frac{-\sqrt{5}}{1 \times \sqrt{5}} = \frac{-\sqrt{5}}{\sqrt{5} \times \sqrt{5}} = -\sqrt{5}) \text{ و } (0, \frac{3}{1})\} \Rightarrow \frac{F}{g} = \{(2, 0) \text{ و } (1, -\sqrt{5}) \text{ و } (0, \frac{3}{1})\}$

$g(x) = \sqrt{4-x^2} \Rightarrow 4-x^2 \geq 0 \Rightarrow 4 \geq x^2 \Rightarrow 2 \geq x \geq -2 \Rightarrow \text{ID}_g = [-2, 2]$

$\text{ID}_f = \{2, 1, 0, 1, 0, 1\} \Rightarrow \text{ID}_f \cap \text{ID}_g = \{2, 1, 0\} \Rightarrow g(2) = \sqrt{4-4} = \sqrt{0} = 0$
 $g(1) = \sqrt{4-1} = \sqrt{3} = \sqrt{3} \quad g(0) = \sqrt{4-0} = 2$

ب) $\frac{g}{f} = \{(2, \frac{0}{2}) \text{ و } (1, \frac{\sqrt{5}}{1-\sqrt{5}}) \text{ و } (0, \frac{3}{1})\} \Rightarrow \frac{g}{f} = \{(1, \frac{\sqrt{5}}{1-\sqrt{5}}) \text{ و } (0, \frac{3}{1})\} \quad \checkmark$

ج) $2F(x) = \{(2, 2) \text{ و } (3, 8) \text{ و } (-5, 4) \text{ و } (1, -4)\}$

د) $F(x)+1 = \{(2, 2) \text{ و } (3, 5) \text{ و } (-5, 3) \text{ و } (1, -1)\}$

هـ) $3F^2(x) + 1 = \{(2, 4) \text{ و } (3, 49) \text{ و } (-5, 13) \text{ و } (1, 13)\}$

و) $F(2x) = \{(1, 1) \text{ و } (\frac{3}{2}, 4) \text{ و } (-\frac{5}{2}, 2) \text{ و } (\frac{1}{2}, -2)\}$

ز) $F-g = \{(1, 4) \text{ و } (2, 2) \text{ و } (3, 2)\}$

ح) $\frac{2F}{g} = \{(2, 4) \text{ و } (3, -2)\}$