

الف) $\frac{x+5}{x^3-10x^2+31x-15} \mid \frac{x-1}{x^2-19x+15}$

$$\begin{array}{r} x^3-10x^2+31x-15 \\ - (x^3-19x^2+15x-15) \\ \hline 9x^2+16x \\ - (9x^2-153x+135) \\ \hline 169x-120 \\ - (169x-120) \\ \hline 0 \end{array}$$

$D_f = \mathbb{R} - \{1, 15\}$

ب) $\frac{x+3}{x^3-10x^2-19x-5} \mid \frac{x+1}{x^2-19x-5}$

$$\begin{array}{r} x^3-10x^2-19x-5 \\ - (x^3-19x^2-5x-5) \\ \hline 9x^2-24x \\ - (9x^2-171x-45) \\ \hline 147x+40 \\ - (147x+147) \\ \hline -107 \end{array}$$

$D_f = \mathbb{R} - \left\{\frac{1}{3}, 1, 2\right\}$

الف) $y = \frac{x+1}{x-\sqrt{3-2x}}$ $\begin{cases} 3-2x \geq 0 \\ x \leq \frac{3}{2} \end{cases}$

$\Rightarrow x \neq \sqrt{3-2x}$
 $\Rightarrow x^2 \neq 3-2x$
 $\Rightarrow x(x+2) \neq 0$
 $\Rightarrow x \neq -2$

$D_f = (-\infty, \frac{3}{2}] - \{-2\}$

ب) $\frac{x+2}{x-\sqrt{3x-2}}$ $\begin{cases} 3x-2 \geq 0 \\ x \geq \frac{2}{3} \end{cases}$

$\Rightarrow x \neq \sqrt{3x-2}$
 $\Rightarrow x^2 \neq 3x-2$
 $\Rightarrow x(x-3) \neq -2$
 $\Rightarrow x \neq 1$

$D_f = [\frac{2}{3}, \infty) - \{1\}$

الف) $\frac{\sin x}{\cos x-1}$ $\cos x \neq 1 \Rightarrow x \neq 2k\pi$

$D_f = \mathbb{R} - \{2k\pi\}$

ب) $\frac{\cos x+1}{\sin x+1}$ $\begin{cases} \sin x \neq -1 \\ x \neq \frac{3\pi}{2} \end{cases}$

$D_f = \mathbb{R} - \left\{2k\pi - \frac{\pi}{2}, 2k\pi - \frac{\pi}{2}\right\}$

ج) $\frac{\tan x+2}{\cot x-1}$ $\cot x \neq 1$

$D_f = \mathbb{R} - \left\{k\pi, k\pi + \frac{\pi}{2}, k\pi + \frac{\pi}{4}\right\}$

د) $\frac{\sin x+1}{\sin x-1}$ $\begin{cases} \sin x \neq 1 \\ x \neq \frac{\pi}{2} \end{cases}$

$D_f = \left\{2k\pi + \frac{\pi}{2}, 2k\pi + \frac{\pi}{2}\right\}$

الف) $x^2-5x+4 > 0$ $\frac{+}{-} \frac{+}{-} \frac{+}{+}$

$(x-1)(x-4) > 0$

$D_f = (-\infty, 1) \cup (4, \infty)$

ب) $x^2-9x+14 < 0$ $\frac{+}{-} \frac{+}{-} \frac{+}{+}$

$(x-2)(x-7) < 0$

$D_f = (2, 7)$

ج) $x^2+9x+4 > 0$ $\frac{-}{+} \frac{-}{-} \frac{-}{+}$

$(x+1)(x+4) > 0$

$D_f = (-\infty, -4] \cup [-1, \infty)$

د) $x^2-14x+49 \leq 0$ $\frac{+}{-} \frac{+}{-} \frac{+}{+}$

$(x-7)^2 \leq 0$

$D_f = [7, 7]$

الف) $\frac{x^2-5x+2}{x-5} > 0 \Rightarrow (x-1)(x-2)$

$\frac{+}{-} \frac{+}{+} \frac{+}{-}$

$D_f = (-\infty, 1) \cup (2, 5)$

ب) $\frac{x^2-1}{x^2-9x+14} > 0 \Rightarrow (x-1)(x+1)$

$\frac{-}{+} \frac{+}{-} \frac{+}{+}$

$D_f = (-\infty, -1] \cup (14, \infty)$

الف) $\sqrt{\frac{a^2 - \varepsilon a + c}{a^2 - 1}} = \sqrt{\frac{(a-c)(a-1)}{a^2 - 1}}$

$\frac{a}{-1} - \frac{c}{1}$

$D_f = [c, \infty)$ ✓

ب) $\sqrt{\frac{a^2 - \varepsilon a + c}{a^2 - 1}}$

$a^2 - 1 \neq 0 \rightarrow a^2 \neq 1 \rightarrow a \neq \pm 1$

$D_f = \mathbb{R} - \{\pm 1\}$ ✓

الز) $\log(a^2 - c a)$ $\frac{a^2 - c a > 0}{a(a-c) > 0}$ $\frac{0}{+1-1+}$

$D_f = (-\infty, 0) \cup (c, \infty)$ ✓

ب) $\log \frac{19-a^2}{|a|-2}$ $\frac{19-a^2 > 0}{|a|-2 > 0}$ $\frac{19 > a^2}{|a| > 2}$ $\frac{19 > a^2}{a > 2 \text{ or } a < -2}$

$D_f = (-\varepsilon - 2) \cup (2 \varepsilon) - \{-c, c\}$ ✓

ج) $\log \frac{a^2 - Va + 12}{10-a}$ $\frac{(a-c)(a-\varepsilon)}{10-a} > 0$ $\frac{a-c}{10-a} > 0$ $a \neq 4$

$D_f = (\varepsilon, 10) - \{4\}$ ✓

د) $\sqrt{a^2 - 11a + 28} + \log \sqrt{c^2 - a^2}$

$\sqrt{(a-7)(a-4)} + \log \sqrt{c^2 - a^2}$

$\frac{\varepsilon}{+1-1+}$ $\frac{a}{a > 7}$ $\frac{a}{a < \varepsilon}$

$D_f = (0, \varepsilon) - \{1\}$ ✓

$y = \frac{a^2 - a}{a^2 + a}$

$\frac{-1}{+1} - \frac{0}{-1} - \frac{1}{-1} +$

$D_f = \mathbb{R} - \{-1, 0\}$ ✓

أ) $\frac{a^2 - a}{a^2 + a}$

	-1	0	1
$a^2 - a$	+	+	+
$a^2 + a$	+	0	+
	+	-	+

$D_f = \mathbb{R} - \{-1, 0\}$ ✓

$y = \sqrt{\frac{a - f(a)}{f(a)}}$

$D_f = [-2, -1] \cup (2, c]$ ✓

$y = a$

	-2	-1	2	c
$a - f(a)$	-	-	+	+
$f(a)$	+	0	-	+
	-	+	-	+

الف) $f(a) + 1 \rightarrow$ ✓

ب) $f(a-1) \rightarrow$ ✓

ج) $f(a) \rightarrow$ ✓

د) $f(a+1) + 2 \rightarrow$ ✓

