

الف) $y = \frac{\sin x - 2}{2 \cos x + 2}$

$2 \cos x + 2 \neq 0$
 $2 \cos x \neq -2 \Rightarrow \cos x \neq -1$
 $\Rightarrow x \neq 180^\circ \Rightarrow Df = \mathbb{R} - \{180^\circ\}$

ب) $y = \frac{\sin x + 3}{\cos x - 1}$

$\cos x - 1 \neq 0 \Rightarrow \cos x \neq 1 \Rightarrow x \neq 0^\circ, 360^\circ$
 $Df = \mathbb{R} - \{0^\circ, 360^\circ\}$

الف) $y = \frac{2 \sin x + 1}{\tan x + 1}$

$\tan x + 1 \neq 0 \Rightarrow \tan x \neq -1$
 $x \neq 135^\circ, 315^\circ \Rightarrow Df = \mathbb{R} - \{135^\circ, 315^\circ\}$

ب) $y = \frac{\cos x + 1}{\cot x - 1}$

$\cot x - 1 \neq 0 \Rightarrow \cot x \neq 1$
 $x \neq 45^\circ, 225^\circ \Rightarrow Df = \mathbb{R} - \{45^\circ, 225^\circ\}$

الف) $\sin y = x^2 - 2$

~~$Df = \{x \mid -1 \leq x^2 - 2 \leq 1\}$~~
 $Df = \{x \mid -1 \leq x^2 - 2 \leq 1\} \Rightarrow -1 \leq x^2 - 2 \leq 1 \Rightarrow 1 \leq x^2 \leq 3$

ب) $y = \arccos(\sqrt{x} - 3)$ $-1 \leq \sqrt{x} - 3 \leq 1 \Rightarrow 2 \leq \sqrt{x} \leq 4$

$Df = \{x \mid 4 \leq x \leq 16\}$

الف) $\cos y = |x| - 3$

$-1 \leq |x| - 3 \leq 1 \Rightarrow 2 \leq |x| \leq 4$
 $Df = [-2, -4] \cup [2, 4]$

ب) $y = \arcsin(x^2 + 3x + 1)$ $-1 \leq x^2 + 3x + 1 \leq 1$

~~$x^2 + 3x + 2 - 1 = (x+2)(x+1) - 1$~~
 $0 \leq (x+2)(x+1) \leq 2$
 $Df = \{x \mid -2 \leq x \leq 0\}$

الف) $y = \log_3 x^2 - 4$

$x^2 - 4 > 0 \Rightarrow x^2 > 4 \Rightarrow x > 2$
 $Df = (2, \infty)$

ب) $y = \log_2 2 - |x|$

$2 - |x| > 0 \Rightarrow 2 > |x|$
 $Df = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

الف) $y = \log_{a-x} a-x$

$a-x > 0 \Rightarrow a > x \quad Df = a > x$

ب) $y = \log_{x+3} x^2-1$

$x^2-1 > 0 \Rightarrow x^2 > 1 \Rightarrow x > 1$
 $Df = x > 1$

الف) $y = \log_{x-3} x^2-5x+3$

$x-3 \neq 0 \Rightarrow x \neq 3$

$x^2-5x+3 = a(x-3)(x-1)$

$\Rightarrow x \neq 3, 1 = 0$

$\frac{x^2-5x+3}{x-3} > 0 \Rightarrow Df = (1, 3) \cup (3, +\infty)$

ب) $y = \log_{x+2} \frac{x+3}{x-2}$

$\frac{x+3}{x-2} > 0$ و $x-2 \neq 0 \Rightarrow x \neq 2$

$Df = (-\infty, -2) \cup (-2, +\infty)$

الف) $y = \sqrt{3 - \log_y (x-2)}$

$3 - \log_y (x-2) \geq 0, x-2 > 0$
 $x > 2$

$Df = 2 < x \leq 11$

$\Rightarrow 3 \geq \log_y (x-2) \Rightarrow 1 \geq x > 2$

ب) $y = \log (2 \log_{\sqrt{x}} x - 1)$

$2 \log_{\sqrt{x}} x - 1 > 0 \Rightarrow 2 \log_{\sqrt{x}} x > 1$
 $\Rightarrow \log_{\sqrt{x}} x > \frac{1}{2}$

$Df = \sqrt{x} < x$

الف) $\frac{x}{x^n+1} = y$

$x^n+1 \neq 0 \Rightarrow x^n \neq -1 \Rightarrow Df = \mathbb{R}$

ب) $y = \frac{x}{x^n-1}$

$x^n-1 \neq 0 \Rightarrow x^n \neq 1 \Rightarrow Df = \mathbb{R} - \{0\}$

ج) $y = \frac{x}{x^n-2}$

$x^n-2 \neq 0 \Rightarrow x^n \neq 2 \Rightarrow Df = \mathbb{R} - \{\frac{1}{2}\}$

د) $y = \frac{x}{x^n-3}$

$x^n-3 \neq 0 \Rightarrow x^n \neq 3 \Rightarrow Df = \mathbb{R} - \{\log_{\sqrt{x}} 3\}$

الف) $y = (x^n+1)!$

$x^n+1 \in \mathbb{N} \Rightarrow x^n \in \mathbb{N}-1$
 $\Rightarrow x \in \frac{\mathbb{N}-1}{n}$

ب) $y = (\frac{x^n-2}{x^n-1})!$

$x^n-1 \neq 0 \Rightarrow x^n \neq 1 \Rightarrow x \neq \pm 1, 0$
 $\frac{x^n-2}{x^n-1} \in \mathbb{N} \Rightarrow x^n-2 \in x^n \mathbb{N}-1 \mathbb{N}$
 $\Rightarrow x^n \in \frac{x^n \mathbb{N}-1 \mathbb{N}+2}{n}$