

$$f(x) = \begin{cases} x^r + rx : x > a \\ ax - f : x \leq a \end{cases} \Rightarrow x = a \Rightarrow f(a) = \begin{cases} a^r + ra \\ ax - f = a^r - f \end{cases}$$

$$\rightarrow a^r + ra = a^r - f \rightarrow ra = -f \rightarrow \boxed{a = -\frac{f}{r}} \quad \checkmark$$

$$g(x) = rx + b \xrightarrow{(r, r)} r = f + b \rightarrow b = -1 \rightarrow f(x) = \frac{x^r + a}{rx - b}$$

$$b = -1 \Rightarrow f(x) = \frac{x^r + a}{rx + 1} \xrightarrow{(r, r)} r = \frac{f + a}{f + 1} \rightarrow | \Delta = a + f \Rightarrow a = -1 \rightarrow b = -1$$

$$f(x) = \frac{x^r + 1}{rx + 1} \Rightarrow f(x) = \frac{x^r + 1}{rx + 1} \rightarrow f(1) = \frac{1^r}{r} = \frac{1}{r} = \boxed{f} \quad \checkmark$$

$$f(x) = \frac{fx + 1}{rx^r + ax + b} \rightarrow D_f = \mathbb{R} - \{-1, f\} \quad \left. \begin{array}{l} \text{عبارت منفرجه دالتي} \\ \text{صفرية} \end{array} \right\} \begin{array}{l} \text{عبارت منفرجه دالتي} \\ \text{صفرية} \end{array}$$

$$\Rightarrow \begin{cases} x = -1 \rightarrow r - a + b = 0 \\ x = f \rightarrow r^r + fa + b = 0 \end{cases} \Rightarrow \Delta a + r^r = 0 \Rightarrow \Delta a = -r^r \Rightarrow a = -\frac{r^r}{\Delta}$$

$$r - a + b = 0 \Rightarrow r + \frac{r^r}{\Delta} = 0 \Rightarrow b = -\frac{r + r^r}{\Delta} \quad f(x) = \frac{fx + 1}{rx^r - \frac{r + r^r}{\Delta}x - \frac{r + r^r}{\Delta}}$$

$$f(x) = \frac{x^r - \frac{r + r^r}{\Delta}}{-fx^r + ax + b} \quad D_f = \mathbb{R} - \{-1\}$$

$$k(x+1)^r = -fx^r + ax + b$$

$$-f(x^r + r(x+1)) = -fx^r + ax + b \Rightarrow -fx^r - \Lambda x - f = -fx^r + ax + b$$

$$\boxed{(-\Lambda) + -f = -1^r} \quad \checkmark$$

$$f(x) = \frac{rx}{(m-1)(x^r + mx + 1)} \quad D_f = \mathbb{R} - \{1\}$$

$\Delta < 0 \Rightarrow$ no real roots

$$x^r + mx + 1 \Rightarrow \Delta < 0 \Rightarrow m^r - f < 0 \Rightarrow m^r < f \Rightarrow \boxed{-r \leq m < r}$$

$$f(x) = \sqrt{f - \frac{1}{x^r}} \Rightarrow f - \frac{1}{x^r} \geq 0 \Rightarrow (r - \frac{1}{x}) \times (r + \frac{1}{x}) \geq 0 \quad \text{①} \quad -r$$

$$\frac{+}{-} \frac{+}{-} \Rightarrow D_f = (-\infty, -\frac{1}{r}] \cup [\frac{1}{r}, +\infty) \quad \checkmark$$

$$f(x) = \sqrt{mx^r + rmx + 1} \Rightarrow mx^r + rmx + 1 \geq 0 \Rightarrow m \geq 0 \quad \text{①} \quad -v$$

mx^r + rmx + 1 ≥ 0 → Δ ≤ 0

$$\rightarrow fm^r - fm \geq 0 \Rightarrow fm(m-1) \geq 0$$

Δ ≤ 0 → Δ ≥ 0 → b^r - 4ac ≥ 0

Δ ≤ 0 → fm^r - fm ≤ 0 → fm(m-1) ≤ 0

$$\frac{0}{+} \frac{1}{-} \Rightarrow m = [1, +\infty) \cup \{0\} \quad \text{①}$$

m > 0 → x^r = 0 → x = 0

Δ ≤ 0 → fm^r - fm ≤ 0 → fm(m-1) ≤ 0

m > 0 → \frac{1}{r} = 1 → m ∈ [0, 1]

Δ ≤ 0 → fm^r - fm ≤ 0 → fm(m-1) ≤ 0

$$f(x) = g(x) \rightarrow f(\frac{1}{r}) = g(\frac{1}{r}) \rightarrow r+k = 1+1 \Rightarrow k=0 \quad -\Delta$$

$$f(x) \rightarrow \frac{fx^r - 1}{rx - 1} \Rightarrow x \neq a \Rightarrow a \rightarrow ra - 1 = 0 \Rightarrow ra = 1 \Rightarrow a = \frac{1}{r} \quad \text{①}$$

$$\left. \begin{array}{l} fx + k \Rightarrow x = \frac{1}{r} \\ a = \frac{1}{r} \\ k = 0 \end{array} \right\} a + k = \frac{1}{r} + 0 = \frac{1}{r} \quad \checkmark$$

$$f(x) = rx + r \quad x = -\frac{r}{r} \rightarrow f(-\frac{r}{r}) = -ra + r \rightarrow f(x) = g(x) \quad -9$$

$$*f(0) = g(0) \Rightarrow \frac{0-f}{0-r} = b \Rightarrow b = -r$$

$$\rightarrow g(x) = rx + b \quad x = -\frac{r}{r} \rightarrow -r + b \xrightarrow{b=-r} g(-\frac{r}{r}) = -f$$

$$g(-\frac{r}{r}) = f(-\frac{r}{r}) = -ra + r = -f \Rightarrow -ra = -f \Rightarrow a = \frac{f}{r}$$

$$a = r, b = -r \Rightarrow a - b = r - (-r) = 2r \quad \text{①} \quad \checkmark$$

دو تابع برابرند → مساوی می‌سازیم → x = r

$$r(a+r)(a-1) = 0 \Rightarrow \begin{cases} g(x) = r + r = f \\ f(x) = ra + ra \end{cases} \Rightarrow f = ra + ra \quad \text{①} \quad -10$$

$$\Rightarrow ra^2 + ra - f = 0 \Rightarrow \begin{cases} a = f/r \\ a = -r \end{cases}$$

جمع ضرب = 0