

$$f(x) = \begin{cases} x^2 + 2x & ; x \geq a \xrightarrow{x=a} a^2 + 2a \\ ax - f & ; x \leq a \xrightarrow{x=a} a^2 - f \end{cases} \rightarrow \begin{cases} a^2 - f = a^2 + 2a \\ \Rightarrow a = -2 \end{cases}$$

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$$f(x) = \frac{x^2 + a}{x - b}, g(x) = 2x + b$$

$(2, 3) \rightarrow$ در هر دو مشترک $f(2) = \frac{4+a}{2-b} = 3, g(2) = 4+b = 3 \Rightarrow b = -1$

$f(1) = ?$

$$f(1) = \frac{1+a}{1-b} = 2 \rightarrow 1+a = 2(1-b) \rightarrow 1+a = 2-2b \rightarrow a = 1-2b$$

$\Delta = -2+a \Rightarrow a = 1$

$$f(x) = \frac{x^2 + 1}{x - 1} \rightarrow f(1) = \frac{1+1}{1-1} = f$$

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$$f(x) = \frac{fx + 1}{2x^2 + ax + b} \quad Df = R - \{-1, f\}$$

$f(1) = ?$

$$f(1) = \frac{f+1}{2+a+b} = -1 \rightarrow f+1 = -2-a-b \rightarrow a+b = -f-1$$

$\frac{-b}{a} = 3 \rightarrow b = -3a$ $\frac{c}{a} = -f \rightarrow c = -fa$

$$\frac{-a}{2} \Rightarrow a = -2 \rightarrow b = 6, c = 2f$$

$$f(x) = \frac{fx + 1}{2x^2 - 4x - 2}$$

$x^2 - 2x - 1 = 0 \rightarrow x = 1 \pm \sqrt{2}$

$$f(1) = \frac{f+1}{-2} = -\frac{f+1}{2}$$

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$$f(x) = \frac{x^2 - \sqrt{2}}{-2x^2 + ax + b} \quad Df = R - \{-1\}$$

$f(1) = ?$

$$f(1) = \frac{1 - \sqrt{2}}{-2 + a + b} = -1 \rightarrow 1 - \sqrt{2} = 2 - a - b \rightarrow a + b = 1 + \sqrt{2}$$

$\frac{-b}{a} = -2, \frac{c}{a} = 1 \rightarrow \frac{b}{-a} = 1 \Rightarrow b = -a$

$$\frac{-a}{-2} = -2 \Rightarrow a = -4 \rightarrow b = 4$$

$a + b = -4 + 4 = 0 \neq 1 + \sqrt{2}$

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$$f(x) = \frac{2x}{(x-1)(x^2 + mx + 1)}$$

$Df = R - \{1\}$

این عبارت ریشتری ضعیف ندارد

این عبارت ریشتری ضعیف دارد $\rightarrow (x-1)^2 = x^2 - 2x + 1 \Rightarrow m = -2$

$\Delta < 0 \rightarrow b^2 - 4ac < 0 \rightarrow m^2 - 4 < 0 \rightarrow m^2 < 4 \rightarrow -2 < m < 2$

$(-2, 2)$

$(-2, 2) \cup \{-1\} = [-2, 2]$

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$$f(x) = \sqrt{x - \frac{1}{x}} \quad x - \frac{1}{x} \geq 0 \quad \frac{x^2 - 1}{x} \geq 0 \quad \frac{x-1}{x} \geq 0 \quad \begin{matrix} x-1 & \neq & 0 \\ x & & + \\ \hline x-1 & & + \\ x & & - \end{matrix} \quad (-\infty, -\frac{1}{x}] \cup [\frac{1}{x}, +\infty)$$

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$$f(x) = \sqrt{mx^2 + 2mx + 1} \quad mx^2 + 2mx + 1 \geq 0 \quad \Delta = 0 \text{ و } m > 0 \quad \Delta = b^2 - 4ac = 4m^2 - 4m$$

$\Delta < 0$ شرط مورد نیاز $m = 0$ طوری نیست: در این صورت عبارت یک تابع ثابتی شود که از خودی آن IR است.

$\Delta = 0 \rightarrow 4m^2 - 4m = 0 \rightarrow m(m-1) = 0 \rightarrow m = 0 \text{ یا } m = 1$

$\Delta < 0 \rightarrow 4m^2 - 4m < 0 \rightarrow m(m-1) < 0 \rightarrow m \in (0, 1)$

$\frac{m}{\Delta} \begin{matrix} 0 & 1 \\ + & - \\ \hline 0 & - \end{matrix} \quad (0, 1) \cup \{1\} = [0, 1]$

$$f(x) = \begin{cases} \frac{x^2 - 1}{x-1} & ; x \neq 1 \\ x+k & ; x = \frac{1}{x} \end{cases} \quad (x-1)(x+1)$$

$g(x) = x+1 \quad x-1 \neq 0 \quad x \neq 1 \quad a+k = \frac{1}{x}$

$x = \frac{1}{x} \rightarrow g(\frac{1}{x}) = x = x+k \Rightarrow k=0$

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$$f(x) = \begin{cases} \frac{9x^2 - 4}{x+2} & ; x \neq -\frac{2}{3} \\ 3ax + 2 & ; x = -\frac{2}{3} \end{cases} \quad (3x-2)(3x+2)$$

$g(x) = 3x+b \Rightarrow b=2 \rightarrow 3x-2 = 3x+b$

$f(-\frac{2}{3}) = -2a+2 = g(-\frac{2}{3}) = 0 \Rightarrow a = \frac{2}{3}$

$a-b = 3 - (-2) = 5$

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$$f(x) = \begin{cases} \frac{x^2 - F}{x-2} & ; x \neq 2 \\ ka^2 + ax & ; x = 2 \end{cases} \quad g(x) = x+2$$

$ka^2 + ax \xrightarrow{x=2} = x+2$

$ka^2 + 2a = F$

$ka^2 + 2a - F = 0$

$a^2 + 2a - 1 = 0$

$(a+1)(a-1) = 0$

$a = -1 \quad \text{یا} \quad a = 1$

$\downarrow \quad \downarrow$

$\boxed{-2} \quad \boxed{1}$

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