

$f(x) = \begin{cases} x^2 + 2x & ; x \geq a \xrightarrow{x=a} a^2 + 2a \\ ax - f & ; x \leq a \xrightarrow{x=a} a^2 - f \end{cases} \rightarrow a^2 - f = a^2 + 2a \Rightarrow a = -2$  ✓

۱

$f(x) = \frac{x^2 + a}{x - b}, g(x) = 2x + b$   
 $(2, 3) \rightarrow f(2) = \frac{4+a}{2-b} = 3, g(2) = 4+b = 3 \Rightarrow b = -1$   
 $f(1) = ?$   
 $1 - 2b = 4 + a \Rightarrow 1 - 2(-1) = 4 + a \Rightarrow 3 = 4 + a \Rightarrow a = -1$   
 $f(x) = \frac{x^2 + 1}{x + 1} \rightarrow f(1) = \frac{1+1}{1+1} = 1$  ✓

۲

$f(x) = \frac{fx + 1}{2x^2 + ax + b}$   $Df = R - \{-1, f\}$   $-\frac{b}{a} = 3$   $\frac{c}{a} = -f$   
 $f(1) = ?$   $2x^2 + ax + b \neq 0$   $\downarrow$   $-\frac{a}{2} \Rightarrow a = -9$   $\frac{b}{2} = -f \Rightarrow b = -1$   
 $f(x) = \frac{fx + 1}{2x^2 - 9x - 1}$   $f(1) = \frac{f}{-11} = -\frac{f}{11}$   
 $x^2 - 9x - f = 0$   $\frac{f(1)+1}{2(1-f)(1+1)} =$  ✓

۳

$f(x) = \frac{x^2 - \sqrt{2}}{-2x^2 + ax + b}$   $Df = R - \{-1\}$   $-2x^2 + ax + b \neq 0$   
 $-\frac{b}{a} = -2, \frac{c}{a} = 1 \rightarrow \frac{b}{-2} = 1 \Rightarrow b = -2$   
 $-\frac{a}{-2} = -2 \Rightarrow a = -4$   $a + b = -4 - 2 = -6$  ✓

۴

$f(x) = \frac{2x}{(x-1)(x^2 + mx + 1)}$   $Df = R - \{1\}$   $x^2 + mx + 1 \rightarrow$  *نباید ریشه داشته باشد*  
 $\Delta < 0 \rightarrow b^2 - 4ac < 0 \Rightarrow m^2 - 4 < 0 \Rightarrow m^2 < 4 \Rightarrow -2 < m < 2$   
 $(-2, 2)$   
 $(-2, 2) \cup \{-1\} = [-2, 2]$  ✓  
 این عبارت ریشتری ضعیف ندارد  
 این عبارت ریشتری ضعیف دارد  $\rightarrow (x-1)^2 = x^2 - 2x + 1 \Rightarrow m = -2$

۵

$$f(x) = \sqrt{x - \frac{1}{x}} \quad x - \frac{1}{x} \geq 0 \quad \frac{x^2 - 1}{x} \geq 0 \quad \frac{x-1}{x} \geq 0 \quad x = \pm \frac{1}{x}$$

2

6

$$f(x) = \sqrt{mx^2 + 2mx + 1} \quad mx^2 + 2mx + 1 \geq 0 \quad \Delta = 0 \text{ و } m > 0 \quad \Delta = b^2 - 4ac = 4m^2 - 4m$$

$\Delta < 0$  شرط مورد نیاز  $m = 0$  طوری نیست؛ در این صورت عبارت یک تابع ثابتی شود که از خودی آن IR است.

$\Delta = 0 \rightarrow 4m^2 - 4m = 0 \rightarrow m(m-1) = 0 \rightarrow m = 0 \text{ یا } m = 1$

$\Delta < 0 \rightarrow 4m^2 - 4m < 0 \rightarrow m(m-1) < 0 \rightarrow 0 < m < 1$

$\frac{m}{\Delta} = \frac{0}{4 \cdot 0 - 4} = \frac{1}{-4} \rightarrow (0, 1) \cup \{1\} = [0, 1]$

2

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & ; x \neq 1 \\ x + k & ; x = \frac{1}{x} \end{cases} \quad g(x) = x + 1 \quad x - 1 \neq 0 \quad x \neq 1$$

$x = \frac{1}{x} \rightarrow g(\frac{1}{x}) = x = x + k \Rightarrow k = 0$

$a + k = \frac{1}{x}$

2

8

$$f(x) = \begin{cases} \frac{9x^2 - 4}{x + 2} & ; x \neq -\frac{2}{3} \\ 3ax + 2 & ; x = -\frac{2}{3} \end{cases} \quad g(x) = 3x + b \quad a - b = 3 - (-2) = 5$$

$\Rightarrow b = 2 \rightarrow 3x - 2 = 3x + b$

$f(-\frac{2}{3}) = -2a + 2 = g(-\frac{2}{3}) = 0 \Rightarrow a = \frac{2}{3}$

2

9

$$f(x) = \begin{cases} \frac{x^2 - F}{x - 2} & ; x \neq 2 \\ ka^2 + ax & ; x = 2 \end{cases} \quad g(x) = x + 2$$

$ka^2 + ax \xrightarrow{x=2} = x + 2$

$ka^2 + 2a = F$

$ka^2 + 2a - F = 0$

$a^2 + 2a - 1 = 0$

$(a+1)(a-1) = 0$

$a = -1 \quad a = 1$

2

10