

$f(x) = \sqrt{x - \frac{1}{x}}$ معادله زیر را برای x حل کنید $\rightarrow x - \frac{1}{x} \geq 0 \rightarrow (x - \frac{1}{x}) \times (x + \frac{1}{x}) \geq 0$
اینجا باید بررسی کنیم که آیا این عبارت در \mathbb{R} است
 $\rightarrow D_f = (-\infty, -\frac{1}{x}] \cup [\frac{1}{x}, +\infty)$ (2)

$f(x) = \sqrt{mx^2 + 2mx + 1}$ این عبارت در \mathbb{R} است $\rightarrow mx^2 + 2mx + 1 \geq 0$
 $\rightarrow \Delta \leq 0$
 $\rightarrow m \geq 0$
 $\rightarrow \Delta \geq 0 \rightarrow b^2 - 4ac \geq 0$
 $\rightarrow 4m^2 - 4m \geq 0 \rightarrow 4m(m-1) \geq 0$
 $\rightarrow m = [1, +\infty) \cup \{0\}$
 $\rightarrow m \in [1, +\infty) \cup \{0\}$
 $\Delta \leq 0 \rightarrow 4m^2 - 4m \leq 0 \rightarrow 4m(m-1) \leq 0 \rightarrow m \in (0, 1]$

$f(x) = g(x) \rightarrow f(\frac{1}{x}) = g(\frac{1}{x}) \rightarrow x + k = 1 + 1 \rightarrow k = 0 \rightarrow k = 0$
 $f(x) = \frac{ax^2 - 1}{x^2 - 1}$; $x \neq a$
 $f(x) = \frac{ax^2 + k}{x^2 + 1}$; $x = \frac{1}{x}$
 $\rightarrow \begin{cases} a = \frac{1}{x} \\ k = 0 \end{cases} \rightarrow a + k = \frac{1}{x} + 0 = \frac{1}{x}$
 $\rightarrow a + k = \frac{1}{x}$ (2)

$f(x) = \frac{ax^2 + k}{x^2 + 1}$ $\rightarrow f(\frac{1}{x}) = \frac{a(\frac{1}{x})^2 + k}{(\frac{1}{x})^2 + 1} = \frac{a + kx^2}{1 + x^2}$
 $g(x) = \frac{bx^2 + c}{x^2 + 1}$
 $f(x) = g(x) \rightarrow \frac{a + kx^2}{1 + x^2} = \frac{bx^2 + c}{x^2 + 1}$
 $\rightarrow a + kx^2 = bx^2 + c$
 $\rightarrow \begin{cases} a = c \\ k = b \end{cases} \rightarrow a - b = c - c = 0$
 $\rightarrow a - b = 0$ (2)

$f(x) = \frac{ax^2 + k}{x^2 + 1}$ $\rightarrow f(\frac{1}{x}) = \frac{a + kx^2}{1 + x^2}$
 $g(x) = \frac{bx^2 + c}{x^2 + 1}$
 $f(x) = g(x) \rightarrow \frac{a + kx^2}{1 + x^2} = \frac{bx^2 + c}{x^2 + 1}$
 $\rightarrow a + kx^2 = bx^2 + c$
 $\rightarrow \begin{cases} a = c \\ k = b \end{cases} \rightarrow a = 1$
 $\rightarrow a = 1$ (2)