۵- ج

۶- ب

۷- ب

۸-

$y = ax^2 + bx + c$

با رأس ۳/۲ (c = ۳/۲)

نقاط $(\beta, 1)$ و $(\beta, -5)$ هم‌عرض هستند پس میانگین طول این دو نقطه طول رأس است

$x_{ent} = \frac{-5+1}{2} = -2 \rightarrow -\frac{b}{2a} = -2 \Rightarrow b = 4a$

$y_{ent} = -\frac{1}{2} \rightarrow f(-2) = -\frac{1}{2} \rightarrow 4a - 2b + c = -\frac{1}{2} \quad \xrightarrow{b=4a}$

$4a - 8a + c = -\frac{1}{2} \rightarrow -4a + c = -\frac{1}{2} \rightarrow -4a + \frac{3}{2} = -\frac{1}{2} \rightarrow a = \frac{1}{2}$

$b = 2 \rightarrow f(1) = \beta \rightarrow a + b + c = \frac{1}{2} + 2 + \frac{3}{2} = 4$ ✓ (۲)

۹- (۲)

	α	β	۰
P	-		+

$a > c$

$\beta > 0$

$36 - 4a > 0$

$36 > 4a$

$9 > a$

$36m^2 - (m+14)m + 1 = 0$

۱۰- ج

$\frac{2}{m} = \ldots$

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}}$

$\frac{1}{\sqrt{\alpha}+\sqrt{\beta}} = \ldots$

$\frac{m+14}{36} + 2 \cdot \frac{2}{\sqrt{m}} = 2$

$900 = m + 14$

$m = 886$

$\frac{1}{\alpha + \beta + 2\sqrt{\alpha\beta}} = 2$

$\frac{m+14}{36} = 2$

$\frac{2}{886}$

$\frac{1}{443}$

سوال ۴

۱) ax^2-ax+a → ۱ ریشه مضاعف → $(x-2)^2=x^2-4x+4$, $a=4$ (I)

۲) ax^2-ax+a → دارای ریشه → $\Delta<0$ → $a^2-4a<0$ → $0<a<4$ (II)

} → $0<a\leq 4$

سوال ۵

$\alpha+\beta=-\frac{-12}{3}=4 \rightarrow \beta=4-\alpha$ (I)

$2\alpha^2+\beta^2-4\alpha=5 \xrightarrow{(I)} 2\alpha^2+(4-\alpha)^2-4\alpha=5 \rightarrow 3\alpha^2-12\alpha+9=0 \rightarrow \begin{cases}\alpha=1\\ \alpha=3\end{cases} \rightarrow a=-9$

$\frac{a}{\alpha_{max}}=\frac{-9}{3}=-3$

سوال ۶

$x_S=b=\frac{(5-2a)+(2a+3)}{2}=a \rightarrow S(a,3)$

$\left.\begin{array}{l}5-2a>0 \rightarrow a<\frac{5}{2}\\ 2a+3>0 \rightarrow a>-\frac{3}{2}\\ a-2>0 \rightarrow a>2\end{array}\right\} \xrightarrow{\text{اشتراک}} a=3 \rightarrow A(4,1), B(1,1)$

$(y-3)=a(x-3)^2 \xrightarrow{(1,1)} (1-3)=a(1-3)^2 \rightarrow a=\frac{-1}{8} \rightarrow (y-3)=\frac{-1}{8}(x-3)^2$

$x=0 \rightarrow (y-3)=\frac{-1}{8}(0-3)^2 \rightarrow y=3-\frac{9}{8} \rightarrow y=\frac{15}{8}$ ⇝ طول = $\frac{1}{8}$

سوال ۷

$\begin{cases}ax^2-ax-b=0\\ a\beta^2-a\beta-b=0\\ 4\beta^2+2\alpha^2-2\beta=15\end{cases}$ $S=\alpha+\beta=1 \rightarrow \alpha=1-\beta$

$4\beta^2+2(1-\beta)^2-2\beta-15=0 \rightarrow 2\beta^2-2\beta+1=0 \rightarrow \beta=\frac{2\pm\sqrt{4-8}}{4}=\frac{1\pm\sqrt{-1}}{2}$

$\alpha-\beta=1-\beta-\beta=1-2\beta \rightarrow 1-2\left(\frac{1\pm\sqrt{-1}}{2}\right)=\pm\frac{2}{\sqrt{5}}$ ⇝ اختلاف = $\frac{2}{\sqrt{5}}$

سوال ۹

$x^2+4x+a=0 \rightarrow \begin{cases}S=\frac{-b}{a}=-4\\ P=\frac{c}{a}=a\end{cases}$ $|\alpha-\beta|=\frac{\sqrt{\Delta}}{|a|}=\sqrt{16-4a}$ $\alpha<\beta \rightarrow \alpha-\beta<0$

$3\alpha^2+2\beta^2=12\sqrt{2}+18 \rightarrow \frac{3}{2}(\alpha^2+\beta^2)+\frac{1}{2}(\alpha^2-\beta^2)=12\sqrt{2}+18 \rightarrow \frac{3}{2}(S^2-2P)+\frac{1}{2}(\alpha+\beta)(\alpha-\beta)=12\sqrt{2}+18$

$\rightarrow \frac{3}{2}(S^2-2P)-\frac{1}{2}S\sqrt{16-4a} \rightarrow \frac{3}{2}(16-2a)-\frac{1}{2}(-4)\sqrt{16-4a}=12\sqrt{2}+18 \rightarrow 9-3a+3\sqrt{16-4a}=12\sqrt{2}+18$

$\begin{cases}9-3a=18 \rightarrow a=1\\ 3\sqrt{16-4a}=12\sqrt{2} \rightarrow 16-4a=32 \rightarrow a=1\end{cases}$

سوال ۱۰

$A=\sqrt{\frac{1}{x_1}}+\sqrt{\frac{1}{x_2}}=5 \rightarrow A^2=\frac{1}{x_1}+\frac{1}{x_2}+2\sqrt{\frac{1}{x_1x_2}}=\frac{x_1+x_2}{x_1x_2}+2\sqrt{\frac{1}{x_1x_2}}=\frac{S}{P}+2\sqrt{\frac{1}{P}}=25$

$\rightarrow m+14+2(4)=25 \rightarrow m+22=25 \rightarrow m=-1$

مینیمم $P=\frac{c}{a}=\frac{2}{m}=\frac{2}{-1}=-2$