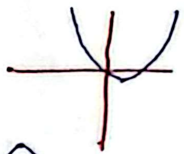
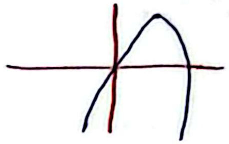
ارتسیا ایزدی دهم پسر β

۱-

الف) $y=3x^2-2x$ ناحیه ۳ $x_s=\frac{2}{6}=\frac{1}{3}$ $y_s=\frac{-\Delta}{4a}=\frac{-4}{12}=-\frac{1}{3}$

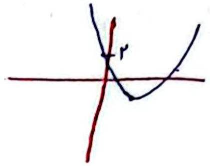


ب) $y=-x^2+4x$ ناحیه ۲ $x_s=\frac{-4}{-2}=2$ $y_s=\frac{-16}{-4}=+4$



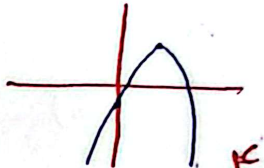
۲-

الف) $y=2x^2-5x+2$ $x_s=\frac{5}{4}$ $y_s=\frac{-(25-16)}{8}=\frac{-9}{8}$



نواحی ۱ و ۲ و ۴

ب) $y=-x^2+4x-1 \Rightarrow x_s=\frac{-4}{-2}=2$ $y_s=\frac{-(16-4)}{-4}=3$



نواحی ۱ و ۳ و ۴

۳-

$x^2-x-3=0$ $s=1$ و $P=-3$ $d=\frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{1+12}}{1}=\sqrt{13}$

الف) $\frac{\alpha+\beta}{\alpha-\beta}=\frac{s}{d}=\frac{1}{\sqrt{13}}$

ب) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(s)^2-2P=1+6=7$

ج) $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha^2\beta-3\alpha\beta^2=(s)^3-\alpha(3\alpha\beta)-\beta(3\alpha\beta)$

$=s^3-(3\alpha\beta)(\alpha+\beta)=1-9\times(-1)=1+9=10$

د) $\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta)=\sqrt{13}(7-3)=4\sqrt{13}$

$$x^2 - ax + a = 0 = k(x-2)^2 \Rightarrow k(x^2 - 4x + 4) = x^2 - ax + a \Rightarrow k = 1 \Rightarrow \overset{-4}{\boxed{a = 4}}$$

1

$$x^2 - ax + a$$

$$\Delta < 0 \Rightarrow a^2 - 4a < 0 \Rightarrow a(a-4) < 0$$

		0		4	
	$+$	$\circ$	$-$	$\circ$	$+$

$$\Rightarrow D_a = \boxed{(0, 4)}$$

2

$$1 \cup 2 \Rightarrow \cancel{a}\ (0, 4]$$

۱۸

۱۸

$P(x) = ۳x^۲ - ۱۲x - a = ۰ \begin{cases} S = ۴ = \alpha + \beta \Rightarrow \beta = ۴ - \alpha \Rightarrow \alpha - ۴ = -\beta \\ P = -\frac{a}{۳} \end{cases}$ $\quad \frac{a}{\alpha - ۵} = \frac{-۹}{۳}$

$۲\alpha + \beta^۲ - ۴\alpha = ۷ \Rightarrow \alpha^۲ + (\alpha^۲ + \beta^۲) - ۴\alpha = ۷ \Rightarrow \alpha(\alpha - ۴) + (۱۶ + \frac{۲a}{۳}) = ۷$

۳ − ۱/۰ ،/۰

$\Rightarrow \alpha(\alpha - ۴) + (۱۶ + \frac{۲\alpha}{۳}) = ۷ \Rightarrow -\alpha\beta + \frac{۲a}{۳} = -۹ \Rightarrow \frac{a}{۳} + \frac{۲a}{۳} = -۹$

$\Rightarrow a = -۹ \overset{P(x)}{\Longrightarrow} ۳x^۲ - ۱۲x + ۹ \overset{\div ۳}{\Longrightarrow} x^۲ - ۴x + ۳ = (x - ۱)(x - ۳) \Rightarrow \alpha = ۳ , \beta = ۱$

-۶

$y_A = y_B = a-2 \Rightarrow x_S = \frac{x_A + x_B}{2} = \frac{\cancel{2a}+3+7-\cancel{2a}}{2} = 5 = b = x_S$

$y_S = b-2 = 5-2 = 3 \qquad \left.\begin{array}{l} 7-2a>0 \Rightarrow a<3/5 \\ a-2>0 \Rightarrow a>2 \end{array}\right\} \xrightarrow{a\in\mathbb{N}} a=3 \Rightarrow \left.\begin{array}{l}(1,1)\\(9,1)\end{array}\right\}$

$\Rightarrow y = kx^2+bx+c \Rightarrow \left.\begin{array}{l} 3 = 25k+5b+c \\ 1 = k+b+c \\ 1 = 81k+9b+c \end{array}\right\} \Rightarrow \left.\begin{array}{l} \overbrace{2 = 24k+4b}^{\times 2} \\ 80k+8b=0 \end{array}\right\} \Rightarrow 32k=-4$

$\Rightarrow k = -\frac{1}{8}$

$9b = \frac{5}{4}$

$\boxed{c = -\frac{1}{8}}$

-۷

$\left.\begin{array}{l} S=1 \\ P=\frac{1}{20}\end{array}\right\} f(x) = ax^2 - ax + \frac{a}{20} \Rightarrow d_{(\text{ریشه‌ها})} = \frac{\sqrt{\Delta}}{|a|}$

$= \frac{\sqrt{a^2-\frac{a^2}{5}}}{|a|} = \frac{\frac{2\cancel{|a|}}{\sqrt{5}}}{\cancel{|a|}} = \frac{2}{\sqrt{5}} \Rightarrow 40\beta^2 + 20\alpha^2 - 20\beta = 17 \xrightarrow{\div 20}$

$(\underbrace{2\beta^2+\alpha^2}_{S^2-2P+\beta^2}) - \beta = \frac{17}{20} \Rightarrow 1 - 2\alpha\beta + \beta\overset{-\alpha}{\cancel{(\beta-1)}} \Rightarrow 1 - 3\alpha\beta = \frac{17}{20}$

$\Rightarrow \boxed{\alpha\beta = \frac{1}{20}}$

-۸

$x_S = \frac{-5+1}{2} = -2 \Rightarrow \text{ext}\left|\begin{array}{l}-2\\-\frac{1}{2}\end{array}\right. \quad y = ax^2+bx+c \qquad c = \frac{3}{2}$

$-\frac{1}{2} = 4a - 2b + \frac{3}{2} \Rightarrow 4a - 2b = 2 \Rightarrow -b = -2 \Rightarrow b=2 ,\ a = \frac{1}{2}$

$\Rightarrow \beta = 4$

$\left.\begin{array}{l} \beta = a+b+\frac{3}{2} \\ \beta = 25a-5b+\frac{3}{2}\end{array}\right\} \Rightarrow 25a-5b = a+b \Rightarrow 24a = 6b \Rightarrow 4a=b$

-۹

$x^2+6x+a=0 \Rightarrow \begin{array}{l} S=6 \\ P=a\end{array} \quad \text{ریشه‌ها} = \frac{-6\pm\sqrt{36-4a}}{2} = -3\pm\sqrt{9-a}$

$\alpha^2 + 2\underbrace{(\alpha^2+\beta^2)}_{S^2-2P} = 12\sqrt{2}+85 \Rightarrow (-3-\sqrt{9-a})^2 + 2(36-2a) = 12\sqrt{2}+85$

$\Rightarrow 9+9-a+6\sqrt{9-a}+72-4a = 12\sqrt{2}+85 \Rightarrow \cancel{6}\sqrt{9-a} = \frac{\cancel{12}}{2}\sqrt{2} \Rightarrow a = 1$

۱۰-

$$36x^2-(m+14)x+1=0 \Rightarrow S=\frac{m+14}{36},\quad P=\frac{1}{36}$$

$$5=\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}=\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}} \Rightarrow \frac{\sqrt{\alpha}+\sqrt{\beta}}{\frac{1}{6}}=5$$

$$\sqrt{\alpha}+\sqrt{\beta}=\frac{5}{6} \xRightarrow{\text{به توان ۲}} \alpha+\beta+2\sqrt{\alpha\beta} \Rightarrow \frac{25}{36}$$

$$\Rightarrow \alpha+\beta+2\sqrt{\frac{1}{36}}=\frac{25}{36} \Rightarrow \alpha+\beta+\frac{1}{3}=\frac{25}{36}$$

$$\Rightarrow \frac{m+14}{36}=\frac{25-12}{36}=\frac{13}{36} \Rightarrow m=-1 \Rightarrow mx^2+3x+2 \Rightarrow P=\frac{2}{m}=-2$$