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$$\begin{aligned} & \frac{-b}{2a} = -1 \rightarrow b = 2a \rightarrow (3, 1) \text{ و } (-1, 9) \rightarrow \begin{cases} 9a + 3b + c = 1 \\ a - b + c = 9 \end{cases} \rightarrow -2a + b \\ & b = 2a \quad a = \frac{-1}{2} \quad b = -1 \quad 9 = \frac{-1}{2} + 1 + c \Rightarrow c = \frac{18+1-2}{2} = \frac{17}{2} \\ & f(x) = \frac{-1}{2}x^2 - x + \frac{17}{2} \end{aligned}$$

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دو مرتبه سه ضلعی داریم

$$\begin{aligned} & x^2 + mx + m + 4 = 0 \quad \Delta \geq 0 \Rightarrow \begin{cases} P \geq 0 \\ S \geq 0 \\ \Delta \geq 0 \end{cases} \rightarrow \begin{aligned} & P = \frac{c}{a} = \frac{m+4}{1} \geq 0 \Rightarrow m \geq -4 \\ & S = \frac{-b}{a} = -m \geq 0 \Rightarrow m \leq 0 \end{aligned} \\ & \Delta \geq 0 \Rightarrow m^2 - 4(m+4) \geq 0 \Rightarrow (m+4)(m-12) \geq 0 \\ & \text{استخراج} \rightarrow m \in (-4, -12) \end{aligned}$$

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$$\begin{aligned} & x_1 + x_2 = \frac{1}{x_1 x_2} \quad S = \frac{1}{p} \quad SP = 1 \\ & S = \frac{-b}{a} / p = \frac{c}{a} \quad S = \frac{-(2m-1)}{3} \quad p = \frac{2-m}{3} \\ & \frac{-(2m-1)}{3} \cdot \frac{2-m}{3} = 1 \quad \frac{-(2m-1)(2-m)}{9} = 1 \\ & -(2m-1)(2-m) = 9 \rightarrow 4m - 2m^2 - 2 + m = 9 \rightarrow -2m^2 + 5m - 11 = 0 \end{aligned}$$

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$$\begin{aligned} & x^2 - x - 4 = 0 \rightarrow S = 1 \quad p = -4 \\ & S = x_1 + x_2 + \frac{1}{x_1} + \frac{1}{x_2} = S(S^2 - 2P) \rightarrow S = \frac{51}{4} \\ & P = \frac{-21}{4} \rightarrow x^2 - \frac{51}{4}x - \frac{21}{4} = 0 \end{aligned}$$

$$x^2 - ax + 2 = 0 \rightarrow \alpha, \beta \quad S = \alpha + \beta = \frac{a}{1} = a$$

$$P = \alpha + \frac{2}{\alpha}$$

$$S = \alpha + \beta = \frac{a}{1} \rightarrow \alpha + \frac{2}{\alpha} = \frac{a}{1} \rightarrow \alpha^2 - a\alpha + 2 = 0$$

$$S = \alpha + \beta = \frac{a}{1} \rightarrow \alpha + \frac{2}{\alpha} = \frac{a}{1} \rightarrow \alpha^2 - a\alpha + 2 = 0$$

$$1 - (-1) = 2$$

$$a > 0$$

$$S > 0 \rightarrow \frac{-(2a+2)}{a} \rightarrow \frac{2}{a} \rightarrow \frac{2}{a} > 0 \rightarrow a < 0$$

$$D > 0 \rightarrow (2a+2)^2 > 0 \rightarrow a \neq -\frac{2}{2}$$

پس به از این نتیجه می‌گیریم که $a < 0$ و $a \neq -1$

$$\frac{-b}{2a} = \frac{2}{-2} = -1 \Rightarrow a = 2$$

$$y = x^2 + ax - 2 \Rightarrow 1 = x^2 + 2x - 2 \Rightarrow x^2 + 2x - 3 = 0 \rightarrow x = -3 \text{ یا } x = 1$$

$$y = -x^2 - 2x + b = 1 \Rightarrow 1 = -1 - 2 + b \Rightarrow b = 4$$

$$ab < 1$$

$$\begin{cases} P = \alpha\beta = \frac{3}{a} \\ S = \alpha + \beta = \frac{-1}{1} \end{cases}$$

$$\left[\frac{ab}{c} \right] \rightarrow -2$$

$$\begin{cases} P = (\alpha + \frac{1}{\alpha})(\beta + \frac{1}{\beta}) = \frac{3}{a} + \frac{b}{1} \rightarrow b = -2 \\ S = \alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta} = \frac{-1}{1} + 1 = 0 \rightarrow a = 2 \end{cases}$$

$$S = \alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta} = \frac{-1}{1} + 1 = 0 \rightarrow a = 2$$

$$x^2 + 4x + m = x^2 + 2x - 2m \rightarrow \boxed{x = -m}$$

$$(-m)^2 + 4(-m) + m = 0 \rightarrow m^2 - 3m = 0 \rightarrow m(m-3) = 0 \rightarrow m = 0 \text{ یا } m = 3$$

$$x^2 + 4x + 5 \rightarrow (x+1)(x+5) \rightarrow x_1 = -1, x_2 = -5$$

$$x^2 + 2x - 10 \rightarrow (x-2)(x+5) \rightarrow x_1 = 2, x_2 = -5$$

$$2 - (-1) = 3$$