

۱

$$\begin{aligned} \frac{-b}{2a} = -1 \rightarrow b = 2a \rightarrow (3, 1) \text{ و } (-1, 9) \rightarrow \begin{cases} 9a + 3b + c = 1 \\ a - b + c = 9 \end{cases} \rightarrow 2a + b \\ b = 2a \quad a = \frac{-1}{4} \quad b = -1 \quad 9 = \frac{-1}{4} + 1 + c \Rightarrow c = \frac{18+1-2}{4} = \frac{17}{4} \\ f(x) = \frac{-1}{4}x^2 - x + \frac{17}{4} \quad \text{✓} \end{aligned}$$

۲

دو دسته سبب دانه دسته

$$\begin{aligned} x^2 + mx + m + 4 = 0 \quad \Delta \geq 0 \Rightarrow \begin{cases} P \geq 0 \\ S \geq 0 \\ \Delta \geq 0 \end{cases} \rightarrow \begin{cases} P = \frac{c}{a} = \frac{m+4}{1} \geq 0 \Rightarrow m \geq -4 \\ S = \frac{-b}{a} = -m \geq 0 \Rightarrow m \leq 0 \end{cases} \\ \Delta = m^2 - 4(m+4) \geq 0 \Rightarrow (m+4)(m-12) \geq 0 \\ \Delta \rightarrow m \in (-4, -1) \quad \text{✓} \end{aligned}$$

۳

$$\begin{aligned} x_1 + x_2 = \frac{1}{x_1 x_2} \quad S = \frac{1}{p} \quad SP = 1 \\ S = \frac{-b}{a} = \frac{2-m}{3} \quad P = \frac{c}{a} = \frac{2-m}{3} \\ \frac{-(2-m)}{3} = \frac{2-m}{3} \Rightarrow -(2-m) = 2-m \Rightarrow -2+m = 2-m \Rightarrow 2m = 4 \Rightarrow m = 2 \\ - (2m-1)(2-m) = 9 \rightarrow 4m - 2m^2 - 2 + m = 9 \rightarrow -2m^2 + 5m - 11 = 0 \end{aligned}$$

۴

$$\begin{aligned} x^2 - x - 1 = 0 \rightarrow S = 1 \quad P = -1 \\ S = x_1^2 + x_2^2 + \frac{1}{x_1} + \frac{1}{x_2} = S(S^2 - 2P) \rightarrow S = \frac{51}{4} \\ P = \frac{-21}{4} \rightarrow x^2 - 8x + P = 0 \quad x^2 - \frac{51}{4}x - \frac{21}{4} = 0 \quad \text{✓} \end{aligned}$$

۵

$$\begin{aligned} (\sqrt[3]{2x} + \frac{1}{\sqrt[3]{2x}} + 1)(\sqrt[3]{2x} - 1) = \sqrt[3]{2x} \rightarrow \frac{(\sqrt[3]{2x} + \sqrt[3]{2x} + 1)(\sqrt[3]{2x} - 1)}{\sqrt[3]{2x}} = \sqrt[3]{2x} \\ \rightarrow x^2 - 1 = 2x \rightarrow x^2 - 2x - 1 = 0 \rightarrow x_1 + x_2 = \frac{-b}{a} = 2 \end{aligned}$$

$$x^2 - ax + 2 = 0 \rightarrow \alpha, \beta \quad S = \alpha + \beta = \frac{a}{1} \\ P = \alpha\beta = \frac{2}{1} \\ S = \alpha + \beta = \frac{a}{1} \rightarrow \alpha + \frac{2}{\alpha} = \frac{a}{1} \rightarrow \alpha^2 + 2 = a\alpha \rightarrow \alpha^2 - a\alpha + 2 = 0 \\ S = \alpha + \beta = \frac{a}{1} \rightarrow \alpha + \frac{2}{\alpha} = \frac{a}{1} \rightarrow \alpha^2 + 2 = a\alpha \rightarrow \alpha^2 - a\alpha + 2 = 0 \\ 1 - (-1) = 2 \checkmark$$

$$a > 0 \\ S > 0 \rightarrow \frac{-(2a+2)}{a} \\ \Delta > 0 \rightarrow (2a+2)^2 > 0 \rightarrow a \neq -\frac{2}{2} \\ \text{استیکان نیست} \\ \text{پس به ازای هیچ مقدری از } a \text{، حکم درست نمی آید}$$

$$\frac{-b}{2a} = \frac{2}{-2} = \frac{-a}{1} \Rightarrow a = 2 \\ \textcircled{1} y = x^2 + ax - 2 \Rightarrow 1 = x^2 + 2x - 2 \Rightarrow x^2 + 2x - 3 = 0 \rightarrow a = 1 \text{ یا } x = -3 \\ \textcircled{2} y = x^2 - 2x + b = 1 \Rightarrow 1 = 1 - 2 + b \Rightarrow b = 2 \\ ab = 2 \checkmark$$

$$\text{مقادیر } \begin{cases} P = \alpha\beta \rightarrow \frac{2}{a} \\ S = \alpha + \beta = \frac{-1}{1} \end{cases} \quad \left[\frac{ab}{1} \right] \rightarrow \frac{-2}{1} \\ \text{مقادیر } \begin{cases} P = (\alpha + \frac{1}{\alpha})(\beta + \frac{1}{\beta}) = \frac{2}{a} + \frac{1}{\alpha\beta} \rightarrow b = -2 \\ S = \alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta} = \frac{-1}{1} + 1 = \frac{1}{1} = \frac{a}{1} \Rightarrow a = 1 \end{cases}$$

$$x^2 + 4x + m = x^2 + 2x - 2m \rightarrow x = -m \\ \begin{cases} (-m)^2 + 4(-m) + m = 0 \rightarrow m^2 - 3m = 0 \rightarrow m(m-3) = 0 \Rightarrow m = 0 \text{ یا } m = 3 \\ \text{مثلاً } \rightarrow x^2 + 4x + 3 \rightarrow (x+1)(x+3) \rightarrow x_1 = -1, x_2 = -3 \\ \rightarrow x^2 + 2x - 6 \rightarrow (x-2)(x+3) \rightarrow x_1 = 2, x_2 = -3 \\ m = (-1) = 2 \checkmark \end{cases}$$