امیدی روشن

(۱) $-\frac{b}{۲a}=-۱ \to b=۲a \mid x=-۱ \to a-\overset{۲a}{b}+c=۹ \to$

$-a+c=۹ \} \; x=۳ \to ۹a+\overset{۲a}{۳b}+\overset{۹+a}{c}=۱ \to ۱۶a=-۸ \to a=-\frac{۱}{۲} \to b=۲\times-\frac{۱}{۲}=-۱$ و

$c=۹-\frac{۱}{۲}=\frac{۱۷}{۲}$

$y=-\frac{۱}{۲}x^۲-x+\frac{۱۷}{۲}$

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(۲) $y=۲x^۲+mx+m+۶=۰ \to$ دو ریشه دارد $\to m^۲-۴(۲m+۱۲)>۰ \to$

$m^۲-۸m-۴۸>۰ \to (m+۴)(m-۱۲)>۰ \to \underset{-۴ \quad\quad ۱۲}{+ \quad - \quad +} \to$

$\mathrm{I}\; m\in(-\infty,-۴)\cup(۱۲,+\infty) \} \; \mathrm{II}\; S=-\frac{m}{۲}>۰ \to m<۰ \} \; \mathrm{III}\; \frac{m+۶}{۲}>۰ \to m>-۶$

$\underline{\mathrm{I},\mathrm{II},\mathrm{III}} \to -۶<m<-۴$

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(۳) $۳x^۲+(۲m-۱)x+۲-m=۰ \to$ دوریشه $\to ۴m^۲+۱-۴m-۴(۶-۳m)>۰$

$۴m^۲+۸m-۲۳>۰ \to m=\frac{-۸\pm\sqrt{۶۴-۴(-۹۲)}}{۸}=\frac{-۸\pm\sqrt{۲^۴(۲^۲+۲۳)}}{۸}=$

$\frac{-۲\pm\sqrt{۲۷}}{۲} \to \underset{-۱-\frac{\sqrt{۲۷}}{۲} \quad\quad -۱+\frac{\sqrt{۲۷}}{۲}}{+ \quad - \quad +} \to \mathrm{I}\; m\in\Big(-\infty,\underbrace{-۱-\frac{\sqrt{۲۷}}{۲}}_{\simeq -\frac{۷}{۲}}\Big)\cup\Big(\underbrace{-۱+\frac{\sqrt{۲۷}}{۲}}_{\simeq\frac{۳}{۲}},+\infty\Big)$

$\mathrm{II}\; \alpha+\beta=\frac{۱}{\alpha\beta} \to \frac{-۲m+۱}{۳}=\frac{۱}{\frac{۲-m}{۳}} \to ۲m^۲-۵m-۷ \to (۲m-۷)(m+۱)=۰$

$-۱ \quad \frac{۷}{۲}$

$\mathrm{I}\cup\mathrm{II} \to \frac{۷}{۲}$

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(۴) $x^۲-x-۴=۰ \to S=۱,\; P=-۴ \} \; x_۲^۳+x_۱^۳+\frac{۱}{x_۲}+\frac{۱}{x_۱}=(x_۱+x_۲)^۳-۳(-۴)(x_۱+x_۲)+\frac{۱}{x_۱x_۲}(x_۱+x_۲)$

$\to ۱+۱۲+-\frac{۱}{۴}=\frac{۵۱}{۴}=S$

$(x_۲^۳+\frac{۱}{x_۲})(x_۱^۳+\frac{۱}{x_۱})=\overset{-۴}{(x_۱x_۲)^۳}+x_۱^۲+x_۲^۲+\frac{۱}{\underset{-۴}{x_۱x_۲}}=-۶۴+۹-\frac{۱}{۴}=$

$\frac{\frac{۱}{x_۱+x_۲}}{\frac{x_۱+x_۲}{-۴}}$

$=-\frac{۲۲۱}{۴} \} \; y=x^۲-\frac{۵۱}{۴}x-\frac{۲۲۱}{۴}$

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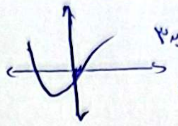
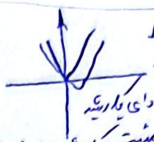
(۵) $\sqrt[۳]{x^۲}=t \to (t+\frac{۱}{t}+۱)(t-۱)=t^۳-\frac{۱}{t} \to \sqrt[۳]{x^۴}-\frac{۱}{\sqrt[۳]{x^۲}}=۲\sqrt[۳]{x}$

$\to \frac{\sqrt[۳]{x^۶-۱}}{\sqrt[۳]{x^۲}}=۲\sqrt[۳]{x} \to x^۲-۱=۲x \to x^۲-۲x-۱=۰ \to S=۲$

(6) $3x^2 - ax + 4 = 0$, $\alpha = 3\beta \to P = \frac{4}{3} \to 3\beta^2 = \frac{4}{3} \Rightarrow \beta^2 = \frac{4}{9} \to \beta \begin{cases} \frac{2}{3} \\ -\frac{2}{3} \end{cases}$

$S \begin{cases} (1)\ \frac{8}{3} = \frac{a}{3} \to a = 8 \\ (2)\ -\frac{8}{3} = \frac{a}{3} \to a = -8 \end{cases} \quad 8 - (-8) = 16 \qquad \alpha \begin{cases} 2 \\ -2 \end{cases}$

(7) $y = ax^2 + (3+2a)x \to \mathrm{I}\ a > 0$, II  دارد ناحیه ۳ شده

 دارای ۲ ریشه مثبت و یک ریشه صفر یا ریشه مضاعف

$\Delta \geq 0 \to 9 + 4a^2 + 12a \geq 0$

$\to (2a+3)^2 \geq 0 \to a \neq -\frac{3}{2}$

(2)

III $S \geq 0 \to \frac{-3-2a}{a} \geq 0 \to -\frac{3}{2} > a \} \mathrm{I}, \mathrm{II}, \mathrm{III} \to \emptyset$

$y = -x^2 - 2x + b$, $y = x^2 + ax - 2 \to -\frac{b}{2a} = \frac{2}{-2} \Rightarrow -\frac{a}{2} \to a = 2$

$\left. \begin{array}{l} 1 = -x^2 - 2x + b \\ 1 = x^2 + 2x - 2 \end{array} \right\} 2 = -2 + b \to b = 4 \} ab = 2 \times 4 = 8$

$2ax^2 + ax - 6 = 0 \to S = -\frac{a}{2a} = -\frac{1}{2}$, $P = -\frac{6}{2} = -3 \}$

$2x^2 - ax + b = 0 \to S = \alpha + \frac{1}{2}, \beta + \frac{1}{2} \Rightarrow -\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = -\frac{(-a)}{2} = a = 1$

$P = (\alpha + \frac{1}{2})(\beta + \frac{1}{2}) = \alpha\beta + \frac{1}{2}(\alpha + \beta) + \frac{1}{4} = \frac{+b}{2} \to -3 + \frac{1}{2}(-\frac{1}{2}) + \frac{1}{4} = \frac{+b}{2}$

$b = -6 \} ab = -6 \times 1 = -6 \} -6 \to \left[\frac{ab}{4}\right] = \left[-\frac{6}{4}\right] = -2$

$x^2 + 6x + m = 0 \to S = -6 \to \overset{\text{ریشه مشترک}}{\alpha} = -6 - \beta \} x^2 + 2x - 3m = 0$

$S = -2 \to \alpha = -2 - \delta \} \mathrm{I}, \mathrm{II} \to -6 - \beta = -2 - \delta \Rightarrow \boxed{\delta - \beta = 4}$