امیری روش **۲۰**

(1) $-\frac{b}{2a} = -1 \to b = 2a \mid x = -1 \to a - b + c = 9 \to$

$-a + c = 9 \} \; x = 3 \to 9a + 3b + c = 21 \to 14a = -7 \to a = -\frac{1}{2} \to b = 2\times -\frac{1}{2} = -1$ و

$c = 9 - \frac{1}{2} = \frac{17}{2}$

$y = -\frac{1}{2}x^2 - x + \frac{17}{2}$ ✓ (2)

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(2) $y = 2x^2 + mx + m + 6 = 0 \to$ دو ریشه دارد $\to m^2 - 4(2m+12) > 0 \to$

$m^2 - 8m - 48 > 0 \to (m+4)(m-12) > 0 \to \underset{-4 \quad 12}{+ \quad - \quad +} \to$

I $m \in (-\infty, -4) \cup (12, +\infty)$ } II $S = -\frac{m}{2} > 0 \to m < 0$ } III $\frac{m+6}{2} > 0 \to m > -6$ (2)

$\underline{\text{I, II, III}} \to -6 < m < -4$ ✓

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(3) $3x^2 + (2m-1)x + 2 - m = 0 \to$ دو ریشه $\to 4m^2 + 1 - 4m - 4(6 - 3m) > 0$

$4m^2 + 8m - 23 > 0 \to m = \frac{-8 \pm \sqrt{64 - 4(-92)}}{8} = \frac{-8 \pm \sqrt{2^4(2^2+23)}}{8} =$

$\frac{-2 \pm \sqrt{27}}{2} \to \underset{\frac{-1-\sqrt{27}}{2} \qquad \frac{-1+\sqrt{27}}{2}}{+ \quad - \quad +} \to$ (I) $m \in \left(-\infty, \underbrace{-1 - \frac{\sqrt{27}}{2}}_{\simeq -\frac{7}{2}}\right) \cup \left(\underbrace{\frac{-1+\sqrt{27}}{2}}_{\simeq \frac{3}{2}}, +\infty\right)$

(II) $\alpha + \beta = \frac{1}{\alpha\beta} \to -\frac{2m-1}{3} = \frac{1}{\frac{2-m}{3}} \to 2m^2 - 5m - 7 \to (2m-7)(m+1) = 0$

$\alpha = -1 \qquad \frac{7}{2}$

$\text{I} \cup \text{II} \to \frac{7}{2}$ ✓ (2)

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(4) $x^2 - x - 4 = 0 \to S = 1, P = -4 \} \; x_2^3 + x_1^3 + \frac{1}{x_2} + \frac{1}{x_1} = (x_1+x_2)^3 - 3(-4)(x_1+x_2) + \frac{x_1+x_2}{x_1x_2}$

$\to 1 + 12 + -\frac{1}{4} = \frac{51}{4} = S$ (2)

$(x_2^3 + \frac{1}{x_2})(x_1^3 + \frac{1}{x_1}) = (x_1x_2)^3 + x_1^2 + x_2^2 + \frac{1}{x_1x_2} = -64 + 9 - \frac{1}{4} = -\frac{221}{4}$

$\frac{x_1+x_2}{x_1x_2} = \frac{1}{-4}$

$= -\frac{221}{4} \} \; y = x^2 - \frac{51}{4}x - \frac{221}{4}$ ✓

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(5) $\sqrt[3]{x^2} = t \to (t + \frac{1}{t} + 1)(t - 1) = t^2 - \frac{1}{t} \to \sqrt[3]{x^4} - \frac{1}{\sqrt[3]{x^2}} = 2\sqrt[3]{x}$

$\to \frac{\sqrt[3]{x^4 - 1}}{\sqrt[3]{x^2}} = 2\sqrt[3]{x} \to x^2 - 1 = 2x \to x^2 - 2x - 1 = 0 \to S = 2$ ✓

(2)

(۶) $۳x^۲ - ax + ۴ = ۰$ ، $\alpha = ۳\beta \to P = \frac{۴}{۳} \to ۳\beta^۲ = \frac{۴}{۳} \to \beta^۲ = \frac{۴}{۹} \to \beta \begin{cases} \frac{۲}{۳} \\ -\frac{۲}{۳} \end{cases}$

$\alpha \begin{cases} ۲ \\ -۲ \end{cases}$

$S \begin{cases} (۱)\ \frac{۸}{۳} = \frac{a}{۳} \to a = ۸ \\ (۲)\ -\frac{۸}{۳} = \frac{a}{۳} \to a = -۸ \end{cases} \Big\}\ ۸ - (-۸) = ۱۶$

(۲)

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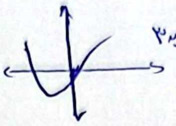
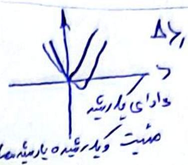
(۷) $y = ax^۲ + (۳ + ۲a)x \to \text{I}\ a > ۰$ ، II





$\Delta \geq ۰ \to ۹ + ۴a^۲ + ۱۲a \geq ۰$

$\to (۲a + ۳)^۲ \geq ۰ \to a \neq -\frac{۳}{۲}$

(۳)

III $S \geq ۰ \to \frac{-۳ - ۲a}{a} \geq ۰ \to -\frac{۳}{۲} > a \}\ \text{I}, \text{II}, \text{III} \to \emptyset$

(۲)

$y = -x^۲ - ۲x + b$ ، $y = x^۲ + ax - ۲ \to -\frac{b}{۲a} = \frac{۲}{-۲} \Rightarrow -\frac{a}{۲} \to a = ۲$

$\left.\begin{array}{l} ۱ = -x^۲ - ۲x + b \\ ۱ = x^۲ + ۲x - ۲ \end{array}\right\}\ ۲ = -۲ + b \to b = ۴ \}\ ab = ۲ \times ۴ = ۸$

(۲)

$۲ax^۲ + ax - ۶ = ۰ \to S = -\frac{a}{۲a} = -\frac{۱}{۲}$ ، $P = -\frac{۶}{۲} = -۳ \}$

$۲x^۲ - ax + b = ۰ \to S = \alpha + \frac{۱}{۲}$ ، $\beta + \frac{۱}{۲} \Rightarrow -\frac{۱}{۲} + \frac{۱}{۲} + \frac{۱}{۲} = -\frac{(-a)}{۲} \Rightarrow a = ۱$

$P = (\alpha + \frac{۱}{۲})(\beta + \frac{۱}{۲}) = \alpha\beta + \frac{۱}{۲}(\alpha + \beta) + \frac{۱}{۴} = \frac{+b}{۲} \to -۳ + \frac{۱}{۲}(-\frac{۱}{۲}) + \frac{۱}{۴} = \frac{+b}{۲}$

$b = -۶ \}\ ab = -۶ \times ۱ = -۶ \}\ -۶ \to \left[\frac{\overset{-۶}{ab}}{۴}\right] = \left[-\frac{۶}{۴}\right] = -۲$

(۲)

$x^۲ + ۶x + m = ۰ \to S = -۶ \to \overset{\text{ریشه مشترک}}{\alpha} = -۶ - \beta \}\ x^۲ + ۲x - ۳m = ۰$

$S = -۲ \to \alpha = -۲ - \delta \}\ \text{I}, \text{II} \to -۶ - \beta = -۲ - \delta \Rightarrow \delta - \beta = ۴$

(۲)