

$$\frac{(1! + 3! + 5! + \dots + 25!)}{70} \times \frac{(2! + 4! + 6! + \dots + 26!)}{6} = 42 \rightarrow 2 \text{ یکان}$$

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$$4 \sqrt{(4 + \sqrt{7})^{-1}} \times \frac{\sqrt{1 + \sqrt{7}}}{4(\sqrt{1 + \sqrt{7}})^2} = 4 \sqrt{\frac{1}{4 + \sqrt{7}} \times (1 + \sqrt{7})^2} = 4 \sqrt{\frac{8 + 2\sqrt{7}}{4 + \sqrt{7}}} = 4 \sqrt{2}$$

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$$\left(\frac{\sqrt{2} + \sqrt{5}}{\sqrt{10} - 2} \right) (\sqrt{3 - \sqrt{5}} - \sqrt{3 + \sqrt{5}}) = \frac{1}{2} (\sqrt{6 - 2\sqrt{5}} - \sqrt{6 + 2\sqrt{5}}) = \frac{1}{2} (\sqrt{(\sqrt{5} - 1)^2} - \sqrt{(\sqrt{5} + 1)^2})$$

$$= \frac{1}{2} (\sqrt{5} - 1 - (\sqrt{5} + 1)) = \frac{1}{2} \times -2 = -1$$

$$\frac{2\sqrt{2} + 3\sqrt{3}}{5 - \sqrt{6}} \times \frac{(5 + \sqrt{6})}{(5 + \sqrt{6})} - 2(\sqrt{5} - 1)^{-1} = \frac{49\sqrt{2} + 10\sqrt{3}}{49} - \left(\frac{2\sqrt{3} + 2}{2} \right) = \sqrt{2} + \sqrt{3} - \sqrt{3} + 1 = \sqrt{2} + 1$$

$$\frac{(3\sqrt{3} - 1)(4 - \sqrt{3})}{4 + \sqrt{3}} \times \frac{(4 - \sqrt{3})}{(4 - \sqrt{3})} = \frac{(3\sqrt{3} - 1)(4 - \sqrt{3})}{13}$$

$$\frac{12\sqrt{3} - 9 - 4 + \sqrt{3}}{13} = \frac{13\sqrt{3} - 13}{13} = \sqrt{3} - 1$$

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$$+ \frac{1}{(2 - \sqrt{3})} \times \frac{(2 + \sqrt{3})}{(2 + \sqrt{3})} = \frac{2 + \sqrt{3}}{(2 - \sqrt{3})(2 + \sqrt{3})} = \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}$$

$$\sqrt{3} - 1 + 2 + \sqrt{3} = 2\sqrt{3} + 1$$

$$(\sqrt{1 + \sqrt{3}} + \sqrt{5 - 1})^2 = 1 + \sqrt{3} + \sqrt{5} - 1 + 2\sqrt{(1 + \sqrt{3})(5 - 1)} = 2\sqrt{3} + 2\sqrt{2} = \sqrt{2\sqrt{3} + 2\sqrt{2}}$$

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$$\Rightarrow \sqrt{2} \sqrt{3 + \sqrt{2}} \Rightarrow \sqrt{3 - \sqrt{2}} = \frac{1}{\sqrt{3 + \sqrt{2}}} \Rightarrow \sqrt{2} \times \sqrt{3 + \sqrt{2}}$$

$$\sqrt{3 + \sqrt{2}} = \sqrt{2}(\sqrt{3} + \sqrt{2}) = \sqrt{6} + 2 \Rightarrow \sqrt{6} + 2 - 2 = \sqrt{6}$$

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$$\sqrt[12]{2^8} \times \sqrt[12]{(3^4 \times 2)^3} \times \sqrt[12]{4^9} = \sqrt[12]{2^8 \times 2^9 \times 3^{12} \times 2^6} = 4 \times 3 = 12$$

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$$\frac{3 \times (1 + 3 + 9 + 27 + 81 + 243)}{2^{n-2} (1 + 2 + 4 + 8 + 16 + 32)} = 52 \Rightarrow \frac{3 \times 364}{2^{n-2} \times 63} = 52 \Rightarrow \frac{3 \times 13 \times 2^2}{2^{n-2} \times 3^2} = 52$$

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$$\Rightarrow \frac{3 \times 13 \times 2^2}{2^{n-2} \times 3^2} = 52 \Rightarrow \frac{3}{3^2} \times \frac{2^2}{2^{n-2}} = 1 \Rightarrow \frac{3}{2^2} \times \frac{2^2}{3} = 1 \Rightarrow n = 2$$

$$(a^2+b^2-2ab)^2 (a^2+b^2+2ab)^2 = (a^2+b^2)^2 - (2ab)^2 = (a^4+b^4+2a^2b^2-4a^2b^2)^2 - 6$$

$$(a^4+b^4-2a^2b^2)^2 \Rightarrow (\sqrt{6}-2+\sqrt{6}+2-2(\sqrt{6}-2)(\sqrt{6}+2))^2 = (2\sqrt{6}-2\sqrt{6-4})^2 = (2\sqrt{6}-2\sqrt{2})^2 = (2\sqrt{6}-2\sqrt{2})^2$$

$$\Rightarrow 4(\sqrt{6}-\sqrt{2})^2 = 4(6+2-2\sqrt{12}) = 4(8-4\sqrt{3}) = 32-16\sqrt{3} = 16(2-\sqrt{3})$$

$$\Rightarrow t(2-\sqrt{3}) = 16(2-\sqrt{3}) \Rightarrow t = 16 \quad \checkmark$$

$$(a+\frac{1}{a}+\sqrt{2})^2 (a+\frac{1}{a}-\sqrt{2})^2 = (a+\frac{1}{a})^2 - (\sqrt{2})^2 \Rightarrow (a^2+\frac{1}{a^2}+2-2) = (a^2+\frac{1}{a^2}+2) = 7$$

$$\Rightarrow \frac{7-4\sqrt{3}+1}{7-4\sqrt{3}} + 2 = 2^t \Rightarrow \frac{8-4\sqrt{3}}{7-4\sqrt{3}} + 2 = 2^t \Rightarrow 7-4\sqrt{3}+7+4\sqrt{3}+2 = 14+2 = 16 = 2^4$$

$$t=4 \quad \checkmark$$

$$A = \sqrt[5]{4^3 \sqrt{16}} \left(\frac{1}{2}\right)^{-\frac{4}{3}} = 2^{\frac{2}{5}} \times 2^{\frac{4}{15}} \times 2^{\frac{4}{3}} = 2^{\frac{6+4+20}{15}} = 2^{\frac{30}{15}} = 2^2 \Rightarrow A=4$$

$$(2 \times 4)^{-\frac{1}{3}} = \sqrt[3]{8}^{-1} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2} \quad \checkmark$$

$$a^{\frac{1}{7}} = 3 \times a^{\frac{15}{7}} \rightarrow a^{\frac{14}{7}} = 3^{-3} \Rightarrow a^2 = 3^{-3} \Rightarrow a = 3^{-\frac{3}{2}} \Rightarrow a = \frac{1}{3\sqrt{3}}$$

$$\Rightarrow \frac{1}{a} - 3 = 3\sqrt{3} - 3 \Rightarrow \frac{\frac{1}{a} - 3}{1+\sqrt{3}} = \frac{3\sqrt{3}-3}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{3\sqrt{3}-9-3+3\sqrt{3}}{1-3}$$

$$\Rightarrow \frac{2(6-3\sqrt{3})}{2} = 6-3\sqrt{3} \quad \checkmark$$

$$\frac{6\sqrt{3}-12}{-2} = \frac{12-6\sqrt{3}}{2} \quad \checkmark$$

$$\sqrt{2+a} - \sqrt{2-4} = 2 \Rightarrow \sqrt{2+a} - 2 = \sqrt{2-4}$$

$$\sqrt{2+a} + \sqrt{2-4} = 2 \Rightarrow \sqrt{2+a} - 2 = 2 \Rightarrow \sqrt{2+a} = 4$$

$$(\sqrt{u+a} + \sqrt{u-r}) (\sqrt{u+a} - \sqrt{u-r}) = (u+a) - (u-r) = a+r$$

$$\rightarrow \frac{a}{r} + r = \frac{a+r}{r}$$

$$\sqrt{u+a} + \sqrt{u-r} = \frac{a}{r} + r \rightarrow \sqrt{u+a} + \sqrt{u-r} - r = \frac{a}{r} + r - r = \frac{a}{r}$$