نام و نام خانوادگی ....۲۵.......... پاسخنامه تشریحی تکلیف شماره ۱۸ ... کلاس ...دهم ریاضی... B

۱

الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4}\cos\frac{13\pi}{4} = -1 + (-\frac{\sqrt{2}}{2} \times -\frac{\sqrt{2}}{2}) = -\frac{1}{2}$ ✓

$\frac{11\pi}{4} = 2\pi + \frac{3\pi}{4} \rightarrow \tan(\frac{11\pi}{4}) = \tan(\frac{3\pi}{4}) = -1$ / $\frac{13\pi}{4} = 2\pi + \frac{5\pi}{4} \rightarrow$

$\frac{15\pi}{4} = 2\pi + \frac{7\pi}{4} \rightarrow \sin\frac{15\pi}{4} = \sin\frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$

$\cos\frac{13\pi}{4} = \cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

ب) $\tan\frac{17\pi}{6}\sin\frac{11\pi}{3} + \cos\frac{10\pi}{3} = (-\frac{\sqrt{3}}{3} \times -\frac{\sqrt{3}}{2}) - \frac{1}{2} = 0$ ✓

$\tan\frac{17\pi}{6} = \tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$

$\sin\frac{11\pi}{3} = \sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos\frac{10\pi}{3} = \cos\frac{4\pi}{3} = -\frac{1}{2}$

(۲)

۲

$\frac{\cos x}{\sin x} = 4 \longrightarrow \cos x = 4\sin x$

$\cot x = 4$

$\frac{3\cos x - \sin x}{\cos x + \sin x}$

$\frac{3(4\sin x) - \sin x}{4\sin x + \sin x} = \frac{11\sin x}{5\sin x} = \frac{11}{5}$ ✓

(۲)

۳

انتهای کمان α در ربع سوم است یعنی $\frac{\sin}{\cos}$ − −

$\sin\alpha = 2\cos\alpha$

$\sin^2 + \cos^2 = 1 \longrightarrow (2\cos)^2 + \cos^2 = 1 \longrightarrow 5\cos^2 = 1 \longrightarrow \cos^2 = \frac{1}{5}$

$\cos = \pm\frac{1}{\sqrt{5}} \Rightarrow \cos = -\frac{1}{\sqrt{5}}$ ✓

منفی چون در ربع ۳ است

(۲)

۴

الف) α در ربع دوم

$\cos x = \sqrt{1 - \sin^2\alpha}$ ... $+\sin$ / $-\cos$

$\cos(\frac{11\pi}{4} + \alpha) = \cos(\frac{3\pi}{4} + \alpha) =$

$\cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha = -\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha = -\frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha)$

$= -\frac{\sqrt{2}}{2}(-\frac{7\sqrt{2}}{10} + \frac{\sqrt{2}}{10}) = -\frac{\sqrt{2}}{2} \times -\frac{6\sqrt{2}}{10} = \frac{12}{20} = \frac{3}{5}$ ✓

(۲)

ب) $\tan x = \frac{1}{7}$ و α در ربع اول : $\sin(\frac{13\pi}{4} + \alpha) = \sin(\frac{5\pi}{4} + \alpha) = \sin\frac{5\pi}{4}\cos\alpha +$

$\sin\alpha\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}\cos\alpha + (-\frac{\sqrt{2}}{2}\sin\alpha) = -\frac{\sqrt{2}}{2}(\sin\alpha + \cos\alpha) = -\frac{\sqrt{2}}{2} \times \frac{40\sqrt{2}}{50} = -\frac{4}{5}$ ✓

$\rightarrow 7\sin\alpha = \cos\alpha \longrightarrow \sin^2 + \cos^2 = 1$; $\sin^2 + 49\sin^2 = 1$

$50\sin^2 = 1$

$\sin^2 = \frac{1}{50} \rightarrow \sin = \pm\frac{1}{\sqrt{50}} = \frac{5\sqrt{2}}{50}$

با توجه به قبل

$\sin\alpha + \cos\alpha = \frac{40\sqrt{2}}{50}$

$\rightarrow \cos = \frac{35\sqrt{2}}{50}$

۵

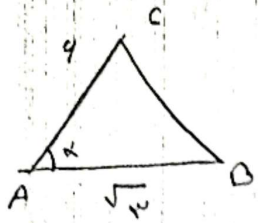
$2\sin^2 x + \cos^2 x = \frac{4}{3} \xrightarrow{\sin^2 x + \cos^2 x = 1} \sin^2 + \underbrace{\sin^2 + \cos^2}_{1} = \frac{4}{3} \longrightarrow \sin^2 x = \frac{1}{3}$

طبق $\sin^2 + \cos^2 = 1 \longrightarrow \cos^2 x = \frac{2}{3}$

$\tan^2 = \frac{\sin^2}{\cos^2} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ ✓

(۲)

۶



$S_{ABC} = \frac{۱}{۲}\sin\alpha \cdot AB \cdot AC \rightarrow ۶ = \frac{۱}{۲}\sin\alpha (۹\sqrt{۲})$

$\sin\alpha = \frac{۶}{۲\sqrt{۲}} = \frac{۱/۵}{\sqrt{۲}} \times \frac{\sqrt{۲}}{\sqrt{۲}} = \frac{۳\sqrt{۲}}{۲} = \frac{\sqrt{۲}}{۲}$

$\sin\alpha = \frac{\sqrt{۲}}{۲} \rightarrow \alpha = \frac{\pi}{۴}$ (min)

$\rightarrow \alpha = \frac{۳\pi}{۴}$ (max)

۲

$\frac{\alpha_{max}}{\alpha_{min}} = \frac{\frac{۳\pi}{۴}}{\frac{\pi}{۴}} = ۳$ ✓

۷

$S = ۶\sqrt{۲}$

$\frac{a}{b} = \frac{۲}{۳}$

$\alpha = ۱۳۵^\circ$

$a = ۲m$

$b = ۳m$

$\rightarrow S = ab\sin\alpha \rightarrow ۶\sqrt{۲} = (۲m)(۳m) \times \frac{۱}{۲}$

$\rightarrow m^۲ = ۸ \rightarrow m = \sqrt{۸} = ۲\sqrt{۲}$

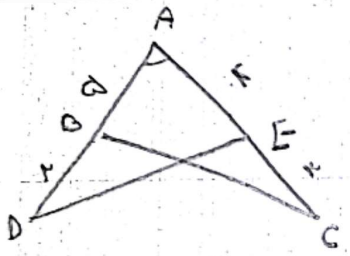
$a = ۲m = ۲ \times ۲\sqrt{۲} = ۴\sqrt{۲}$

$b = ۳m = ۳ \times ۲\sqrt{۲} = ۶\sqrt{۲}$

$P = ۲(a+b) = ۲(۴\sqrt{۲} + ۶\sqrt{۲}) = ۲(۱۰\sqrt{۲}) = ۲۰\sqrt{۲}$ ✓

۲

۸



$S_{ABC} - S_{ADE} = ۱/۷۵$

$S_{ABC} = \frac{۱}{۲} \times ۵ \times ۷ \times \sin\hat{A} = ۱۷/۵\sin\hat{A}$

$S_{ADE} = \frac{۱}{۲} \times ۴ \times ۷ \times \sin\hat{A} = ۱۴\sin\hat{A}$

$\rightarrow ۲/۵\sin\hat{A} = ۱/۷۵$

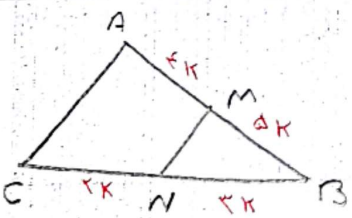
$\Rightarrow \sin\hat{A} = \frac{۱}{۲}$

$\hat{A} = ۳۰^\circ$

$\tan\hat{A} = \tan ۳۰^\circ = \frac{\sqrt{۳}}{۳}$ ✓

۲

۹



$۲\overrightarrow{BN} = ۲\overrightarrow{NC} \rightarrow \frac{BN}{NC} = \frac{۲}{۱} \rightarrow \begin{cases} BN = ۲K \\ NC = ۲K \end{cases}$

$S_{ABC} = ۲S_{BMN}$

$\frac{S_{ABC}}{S_{BMN}} = \frac{\frac{۱}{۲} \times AB \times BC \times \sin\hat{B}}{\frac{۱}{۲} \times BM \times BN \times \sin\hat{B}} = ۲ \rightarrow \frac{۵AB}{۲BM} = ۲$

$\frac{BM}{AB} = \frac{۵K}{۹K} \rightarrow \frac{BM}{AM} = \frac{۵K}{۴K} = ۱/۲۵$ ✓

۲

۱۰

$\frac{۱}{\sqrt{\cos^۲\alpha}} - \tan\alpha = \frac{۱+\sin\alpha}{|\cos\alpha|} \rightarrow \frac{۱}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{۱+\sin\alpha}{|\cos\alpha|}$

$\rightarrow -\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|}$

$\cos > ۰ \rightarrow -\frac{\sin\alpha}{\cos\alpha} = \frac{\sin}{\cos} \rightarrow \sin\alpha = ۰ \rightarrow \alpha = ۲\pi$

$\cos < ۰ \rightarrow -\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{-\cos\alpha} \rightarrow -\tan\alpha = -\tan\alpha$

$\alpha = (\frac{\pi}{۴}, \frac{۳\pi}{۲})$

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{۱}{\cot\alpha} = \frac{\sin\alpha}{\cos\alpha} \rightarrow \frac{|\sin\alpha|}{\cos\alpha} = \frac{-\sin\alpha}{\cos\alpha} \Rightarrow |\sin\alpha| = -\sin\alpha$

$\sin\alpha < ۰$ ✓

۲

ربع سوم