نام و نام خانوادگی: محمدرضا انصاری — پاسخنامه تشریحی تکلیف شماره ۱۱ — کلاس: دهم ب B

۱

الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4} \times \cos\frac{13\pi}{4} = -1 + (-\frac{\sqrt2}{2} \times -\frac{\sqrt2}{2}) = -\frac{1}{2}$

$\frac{11\pi}{4} = 2\pi + \frac{3\pi}{4} \Rightarrow \tan(\frac{11\pi}{4}) = \tan(\frac{3\pi}{4}) = -1$

$\frac{15\pi}{4} = 2\pi + \frac{7\pi}{4} \Rightarrow \sin\frac{15\pi}{4} = \sin\frac{7\pi}{4} = -\frac{\sqrt2}{2}$ / $\frac{13\pi}{4} = 2\pi + \frac{5\pi}{4} \Rightarrow \cos\frac{13\pi}{4} = -\frac{\sqrt2}{2}$

ب) $\tan\frac{17\pi}{6}\,\sin\frac{11\pi}{3} + \cos\frac{10\pi}{3} = (-\frac{\sqrt3}{3} \times -\frac{\sqrt3}{2}) - \frac{1}{2} = 0$ $\quad (\frac{3}{6} = \frac12)$

$\sin\frac{11\pi}{3} = \sin\frac{5\pi}{3} = -\frac{\sqrt3}{2}$ / $\cos\frac{10\pi}{3} = \cos\frac{4\pi}{3} = -\frac{1}{2}$

$\tan\frac{17\pi}{6} = \tan\frac{5\pi}{6} = -\frac{\sqrt3}{3}$

۲

$\frac{\cos x}{\sin x} = 4 \Rightarrow \cos x = 4\sin x$

$\frac{3\cos x - \sin x}{\cos x + \sin x} = \frac{3(4\sin x) - \sin x}{4\sin x + \sin x} = \frac{11\sin x}{5\sin x} = \frac{11}{5}$

۳

انتهای کمان در ربع سوم ⇐ $-\cos$ و $-\sin$

$\sin\alpha = 2\cos\alpha$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow (2\cos)^2 + \cos^2 = 1 \Rightarrow 5\cos^2 = 1 \Rightarrow \cos^2 = \frac{1}{5} \Rightarrow$

$\cos x = \pm\frac{1}{\sqrt5}$ (غ ق ق) $\Rightarrow \cos x = -\frac{1}{\sqrt5}$

۴

الف) α در ربع دوم ⇐ $\sin +$ ، $\cos -$ و $\sin\alpha = \frac{\sqrt2}{10}$: $\cos(\frac{11\pi}{4} + \alpha) = ?$

$\cos(\frac{11\pi}{4} + \alpha) = \cos(\frac{3\pi}{4} + \alpha) \Rightarrow \cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha = -\frac{\sqrt2}{2}\sin\alpha - \frac{\sqrt2}{2}\cos\alpha$

$-\frac{\sqrt2}{2} \times -\frac{6\sqrt2}{10} = \frac{12}{20} = \frac{3}{5} \Leftarrow -\frac{\sqrt2}{2}(-\frac{7\sqrt2}{10} + \frac{\sqrt2}{10}) \Leftarrow -\frac{\sqrt2}{2}(\sin\alpha + \cos\alpha) \Leftarrow$

ب) $\tan\alpha = \frac{1}{7}$ و α در ربع اول : $\sin(\frac{13\pi}{4} + \alpha) = \sin(\frac{5\pi}{4} + \alpha) =$

$7\sin\alpha = \cos\alpha$

$\sin\frac{5\pi}{4}\cos\alpha + \sin\alpha\cos\frac{5\pi}{4} = -\frac{\sqrt2}{2}\cos\alpha + (-\frac{\sqrt2}{2}\sin\alpha) = -\frac{\sqrt2}{2}(\sin\alpha + \cos\alpha) = -\frac{\sqrt2}{2} \times \frac{40\sqrt2}{50} = -\frac{4}{5}$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \sin^2\alpha + 49\sin^2\alpha = 1 \Rightarrow 50\sin^2\alpha = 1$

$\sin^2 = \frac{1}{50} \Rightarrow \sin = \frac{1}{\sqrt{50}} = \frac{5\sqrt2}{50}$

$\cos = \frac{35\sqrt2}{50}$

$\sin\alpha + \cos\alpha = \frac{40\sqrt2}{50}$

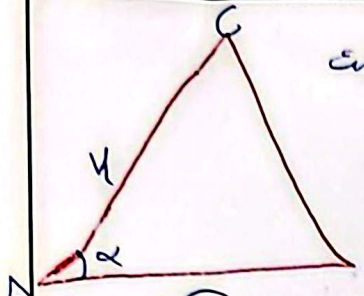
۵

$2\sin^2 x + \cos^2 x = \frac{4}{3}$ ، $\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3}$

$\sin^2 x = \frac{1}{3} \Rightarrow \cos^2 x = \frac{2}{3}$

$\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\frac13}{\frac23} = \frac{1}{2}$

$\sqrt{3} \times 4 = 4\sqrt{3}$



$S_{ABC} = \frac{1}{2}\sin\alpha \times AB \times AC \Rightarrow 6{,}0 = \frac{1}{2}\sin\alpha(4\sqrt{3})$

$\sin\alpha = \frac{6{,}0}{2\sqrt{3}} = \frac{1{,}0}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \frac{\sqrt{3}}{2}$ ۶

$\sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \frac{\pi}{3}$ (min), $\alpha = \frac{2\pi}{3}$ (max)

$\frac{\alpha_{max}}{\alpha_{min}} = \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = 2$

$S = 6r$, $\frac{a}{b} = \frac{2}{3}$, $\alpha = 120^\circ$ $\Rightarrow$ $a = 2m$, $b = 3m$

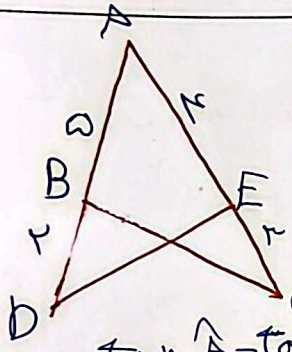
$S = ab\sin\alpha \Rightarrow 6r = \underbrace{(2m)(3m) \times \frac{1}{2}}_{3m^2}$

$\Rightarrow m^2 = 18 \Rightarrow m = \sqrt{18} = 3\sqrt{2}$ ۷

$a = 2m \Rightarrow 2 \times 3\sqrt{2} = 6\sqrt{2}$

$b = 3m \Rightarrow 3 \times 3\sqrt{2} = 9\sqrt{2}$

$P = 2(a+b) = 2(6\sqrt{2} + 9\sqrt{2}) = 2(15\sqrt{2}) = 30\sqrt{2}$



$S_{ABC} - S_{ADE} = 17{,}5$

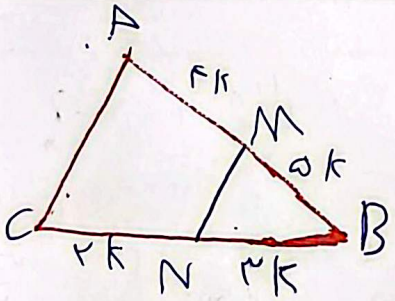
$S_{ABC} = \frac{1}{2} \times 5 \times 7 \times \sin\hat{A} = 17{,}5\sin\hat{A}$

$S_{ADE} = \frac{1}{2} \times 3 \times 7 \times \sin\hat{A} = 10{,}5\sin\hat{A}$ ۸

$7{,}5\sin\hat{A} = 17{,}5$

$\sin\hat{A} = \frac{1}{2} \Rightarrow \hat{A} = 30^\circ$

$\tan\hat{A} = \tan 30^\circ = \frac{\sqrt{3}}{3}$



$2BN = 2NC \Rightarrow \frac{BN}{NC} = \frac{2}{2} \Rightarrow BN = 2k$

$NC = 2k$

$S_{ABC} = 2S_{BMN}$ $\frac{5AB}{2BM} = 2$ ۹

$\frac{S_{ABC}}{S_{BMN}} = \frac{\frac{1}{2} \times AB \times BC \times \sin\hat{B}}{\frac{1}{2} \times BM \times BN \times \sin\hat{B}} = 2$

$\frac{BM}{AB} = \frac{8k}{9k}$

$\frac{BM}{AM} = \frac{8k}{12k} = 1{,}25$

$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1+\sin\alpha}{|\cos\alpha|}$

$\Rightarrow -\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|}$

$\cos\alpha > 0 \Rightarrow -\frac{\sin\alpha}{\cos\alpha} = \frac{\sin}{\cos} \Rightarrow \sin = 0 \Rightarrow \alpha = 0$

$\cos\alpha < 0 \Rightarrow -\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{-\cos\alpha} \Rightarrow -\tan = -\tan$

$\frac{|\sin\alpha|}{\cos\alpha} = \frac{-1}{\cot\alpha}$ $\xrightarrow{\frac{\sin\pi}{\cos\pi}}$ $\frac{|\sin|}{\cos} = \frac{-\sin}{\cos} \Rightarrow |\sin| = -\sin$

$\sin\alpha \leq 0 \Rightarrow$ (II) $\alpha = (\pi, 2\pi)$ (I)

I ∩ II = ناحیه سوم α انتهای کمان