۱۸,۷۵

۱- الف) $\tan\frac{11\pi}{6}+\sin\frac{15\pi}{6}\cos\frac{12\pi}{6}=\tan 330^\circ+\sin 315^\circ\times\cos 225^\circ$

$=-1+\left(\frac{-\sqrt{2}}{2}\times\frac{-\sqrt{2}}{2}\right)=0$ یعنی $-\tan\frac{\pi}{4}+(-\sin\frac{\pi}{4})(-\cos\frac{\pi}{4})=-1\left(-\frac{\sqrt{2}}{2}\right)^2=-\frac{1}{2}$

ب) $\tan\frac{17\pi}{6}\sin\frac{11\pi}{3}+\cos\frac{10\pi}{3}=\tan 150^\circ\times\sin 300^\circ+\cos 240^\circ$

$=-\frac{1}{\sqrt{3}}\times\frac{-\sqrt{3}}{2}+\frac{-1}{2}=0$ ✓ (۱)

۲-

$\cot\alpha=2$ $\quad 1+\cot^2\alpha=\frac{1}{\sin^2\alpha}\Rightarrow 1+4=\frac{1}{\sin^2\alpha}\Rightarrow\sin^2\alpha=\frac{1}{5}$

$\cos^2\alpha+\sin^2\alpha=1\Rightarrow\cos^2\alpha=\frac{4}{5}\Rightarrow\sin\alpha=\frac{1}{\sqrt{5}}$ و $\cos\alpha=\frac{2}{\sqrt{5}}$

$\frac{3\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}=\frac{\frac{6}{\sqrt{5}}-\frac{1}{\sqrt{5}}}{\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{5}}}=\frac{\frac{5}{\sqrt{5}}}{\frac{3}{\sqrt{5}}}=\frac{5}{3}$ ✓ (۲)

۳-

$\sin\alpha=2\cos\alpha$ $\quad \sin^2\alpha+\cos^2\alpha=1\Rightarrow(2\cos\alpha)^2+\cos^2\alpha=1$

$\Rightarrow 5\cos^2\alpha=1\Rightarrow\cos^2\alpha=\frac{1}{5}$ و $\cos\alpha=-\frac{1}{\sqrt{5}}$ ✓ (۲)

۴- الف)

$\frac{11\pi}{4}=270^\circ+135^\circ\Rightarrow\sin\frac{11\pi}{4}=\sin 135^\circ$ و $\cos\frac{11\pi}{4}=\cos 135^\circ$

$\sin^2\alpha+\cos^2\alpha=1\Rightarrow\frac{2}{100}+\cos^2\alpha=1\Rightarrow\cos\alpha=\frac{-\sqrt{98}}{10}$

$\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos 135^\circ\times\cos\alpha-\sin 135^\circ\times\sin\alpha=\frac{-\sqrt{2}}{2}\times\frac{-\sqrt{98}}{10}$

$-\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}=\frac{14}{20}=\frac{12}{20}=\frac{12}{5}$ الف ✓ (۲)

ب) $1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\Rightarrow\frac{25}{49}=\frac{1}{\cos^2\alpha}\Rightarrow\cos^2\alpha=\frac{49}{25}\Rightarrow\cos=\frac{7}{\sqrt{50}}$

$\sin\alpha=\frac{1}{\sqrt{50}}$

$\sin\left(\frac{13\pi}{4}+\alpha\right)=\sin 225^\circ\times\cos\alpha+\sin\alpha\times\cos 225^\circ=\frac{-\sqrt{2}}{2}\times\frac{7}{5\sqrt{2}}+\frac{-\sqrt{2}}{2}\times\frac{1}{5\sqrt{2}}$

$=-\frac{7}{10}+\left(-\frac{1}{10}\right)=-\frac{8}{10}=-\frac{4}{5}$ ب ✓

۵-

$$۲\sin^۲\alpha + \cos^۲\alpha = \frac{۶}{۳} \Rightarrow \sin^۲\alpha = \frac{۱}{۳} \text{ و } \cos^۲\alpha = \frac{۲}{۳}$$

$$\tan^۲\alpha = \frac{\sin^۲\alpha}{\cos^۲\alpha} = \frac{\frac{۱}{۳}}{\frac{۲}{۳}} = \frac{۱}{۲}$$ ✓ (۲)

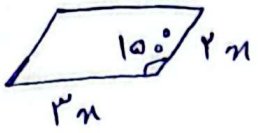
۶-

$$S_{\hat{ABC}} = \frac{۱}{۲} \times ۶ \times \sqrt{۲} \times \sin\alpha = ۴\sqrt{۲} \times \sin\alpha$$

$$\Rightarrow \sin\alpha = \frac{\sqrt{۳}}{۲} \qquad \alpha = ۱۲۰^\circ \text{ یا } ۶۰^\circ$$

$$\frac{۱۲۰}{۶۰} = ۲$$ ✓ (۴)

۷-



$$\frac{۱}{۲} \times ۲x \times ۳x \times \sin ۱۵۰^\circ = ۶\sqrt{۳} = \frac{۳}{۴}x^۲ \times \frac{۱}{۲}$$

$$\Rightarrow x^۲ = ۱۸ \Rightarrow x = \sqrt{۱۸}$$

$$\text{محیط} = ۱۰ \times \sqrt{۱۸} = ۱۰\sqrt{۱۸} = ۳۰\sqrt{۲}$$ ✓ (۴)

۸-

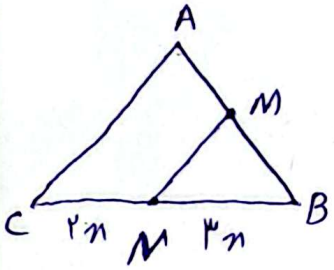
$$S_{\hat{ADE}} = \frac{۱}{۲} \times ۷ \times ۱۲ \times \sin\hat{A}$$

$$S_{\hat{ABC}} = \frac{۱}{۲} \times ۷ \times ۵ \times \sin\hat{A}$$

$$\frac{۳۵}{۲}\sin\hat{A} - \frac{۲۸}{۲}\sin\hat{A} = ۱۰/۵ = \frac{۷}{۲}\sin\hat{A}$$

$$\Rightarrow ۳۰/۵ = ۷\sin\hat{A} \Rightarrow \hat{A} = ۳۰^\circ \qquad \tan ۳۰^\circ = \frac{۱}{\sqrt{۳}}$$ ✓ (۲)

۹-



$$\frac{S_{\hat{ABC}}}{S_{\hat{BMN}}} = \frac{\frac{۱}{۲} AB \times ۵x \times \sin\hat{B}}{\frac{۱}{۲} BM \times ۲x \times \sin\hat{B}} = \frac{۵AB}{۲BM} = ۳$$

$$\Rightarrow \frac{AB}{BM} = \frac{۹}{۵} \Rightarrow \frac{BM}{AB} = \frac{۵}{۹}$$ (۱/۷۵)

$$BM = ۵x,\ AM = ۴x \rightarrow \frac{BM}{AM} = \frac{۵}{۴}$$

۱۰- در ناحیه سوم

$$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} = -\tan\alpha \Rightarrow \sin\alpha < 0$$

(۲)

$$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \frac{1+\sin\alpha}{|\cos\alpha|} = \tan\alpha$$

$$= \frac{-\sin\alpha}{|\cos\alpha|} = \tan\alpha \xrightarrow{\sin\alpha<0} \frac{|\sin\alpha|}{|\cos\alpha|} = \tan\alpha \Rightarrow \cos\alpha < 0$$