۱۰

$aq^۲ = \sqrt{aq^۳} \to a^۲q^۴ = aq^۳ \to aq = ۱ \to q = \frac{۱}{a}$

$a_۵ = aq^۴ = ۲۷ \to a \times \frac{۱}{a^۴} = a^{-۳} = ۲۷ \to a = \frac{۱}{۳}$

$q = ۳$

$\frac{۱}{۲} - \frac{۱}{۳} = \frac{۱}{۶}$

تکلیف ۱۸ — به نام خدا — لیلا رمضانی هم‌پیر B

۱) الف) $\tan\frac{۱۱\pi}{۴} + \sin\frac{۱۵\pi}{۴}\cos\frac{۱۳\pi}{۴}$

$\to \tan(۲\pi + \frac{۳\pi}{۴}) + \sin(۳\pi + \frac{۳\pi}{۴})\cos(۳\pi + \frac{\pi}{۴})$

$\to \tan ۱۳۵^\circ + \sin ۳۱۵^\circ \times \cos ۲۲۵^\circ = -۱ + (-\frac{\sqrt{۲}}{۲} \times -\frac{\sqrt{۲}}{۲}) = -\frac{۱}{۲}$

ب) $\tan\frac{۱۷\pi}{۶}\sin\frac{۱۱\pi}{۳} + \cos\frac{۱۰\pi}{۳}$

$\to \tan(۲\pi + \frac{۵\pi}{۶})\sin(۳\pi + \frac{۲\pi}{۳}) + \cos(۳\pi + \frac{\pi}{۳})$

$\to \tan(۱۵۰^\circ) \times \sin ۳۰۰^\circ + \cos ۲۴۰^\circ = -\frac{\sqrt{۳}}{۳} \times -\frac{\sqrt{۳}}{۲} + -\frac{۱}{۲} = ۰$

۲) $\cot x = ۴ \to \frac{\cos x}{\sin x} = ۴ \to \cos x = ۴\sin x$

$\frac{۳\cos x - \sin x}{\cos x + \sin x} \xrightarrow{\cos x = ۴\sin x} \frac{۱۲\sin x - \sin x}{۴\sin x + \sin x} = \frac{۱۱\sin x}{۵\sin x} = \frac{۱۱}{۵}$

۳) $\sin\alpha = ۲\cos\alpha$ — α ناحیه ۳

$\to \sqrt{۱-\cos^۲\alpha} = ۲\cos\alpha \to ۱-\cos^۲\alpha = ۴\cos^۲\alpha \to \cos^۲\alpha = \frac{۱}{۵}$

$\to \cos\alpha = \pm\frac{\sqrt{۵}}{۵}$ —(α ناحیه ۳)→ $\cos\alpha = -\frac{\sqrt{۵}}{۵}$

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۴) α در ربع دوم

الف) $\sin\alpha = \frac{\sqrt{۲}}{۱۰}$ $\quad \sin^۲\alpha + \cos^۲\alpha = ۱ \rightarrow \frac{۲}{۱۰۰} + \cos^۲\alpha = ۱ \rightarrow \cos\alpha = -\frac{\sqrt{۹۸}}{۱۰}$

$\cos\left(\frac{۱۱\pi}{۴} + \alpha\right) = \cos\left(۱۳۵^\circ + \alpha\right) = \cos ۱۳۵^\circ \times \cos\alpha - \sin ۱۳۵^\circ \times \sin\alpha$

$= -\frac{\sqrt{۲}}{۲} \times \frac{۷\sqrt{۲}}{۱۰} - \frac{\sqrt{۲}}{۲} \times \frac{\sqrt{۲}}{۱۰} = \frac{۷}{۱۰} - \frac{۱}{۱۰} = \frac{۶}{۱۰} = \boxed{\frac{۳}{۵}}$

ب) α در ربع اول

$\tan\alpha = \frac{۱}{۷} \rightarrow \tan^۲\alpha + ۱ = \frac{۱}{\cos^۲\alpha} \rightarrow \frac{۵۰}{۴۹} = \frac{۱}{\cos^۲\alpha} \rightarrow \cos\alpha = \frac{۷}{\sqrt{۵۰}}$

$\tan\alpha = \frac{\sin\alpha}{\cos} = \frac{۱}{۷} \rightarrow \frac{\sin\alpha}{\frac{۷}{\sqrt{۵۰}}} = \frac{۱}{۷} \rightarrow \sin\alpha = \frac{۱}{\sqrt{۵۰}}$

$\sin\left(۱۴\pi + \alpha\right) = \sin\left(\left(۳۶\pi + \frac{\pi}{۴}\right) + \alpha\right) = \sin\left(۲۲۵^\circ + \alpha\right)$

$= \sin ۱۳۵^\circ \times \cos\alpha + \cos ۱۳۵^\circ \times \sin\alpha = \frac{-\sqrt{۲}}{۲} \times \frac{۷}{\sqrt{۵۰}} + \frac{-\sqrt{۲}}{۲} \times \frac{۱}{\sqrt{۵۰}}$

$= \frac{-۷}{۱۰} - \frac{۱}{۱۰} = \frac{-۸}{۱۰} = \boxed{-\frac{۴}{۵}}$

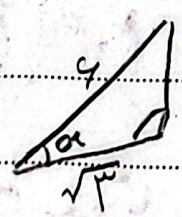
پ) $۲\sin^۲ x + \cos^۲ x = \frac{۴}{۳} \rightarrow \sin^۲ x + \sin^۲ x + \cos^۲ x = \frac{۴}{۳}$

$\rightarrow \sin^۲ x = \frac{۱}{۳} \quad \rightarrow \sin^۲ x + \cos^۲ x = ۱ \rightarrow \frac{۱}{۳} + \cos^۲ x = ۱$

$\rightarrow \cos^۲ x = \frac{۲}{۳} \quad \tan^۲ x + ۱ = \frac{۱}{\cos^۲ x} \rightarrow \tan^۲ x = \frac{۱}{\frac{۲}{۳}} - ۱$

$\rightarrow \tan^۲ x = \boxed{\frac{۱}{۲}}$

ت) $\frac{۱}{۲}\sin\alpha \sqrt{۳} \times ۶ = ۴,۵$



$\rightarrow \sin\alpha = \frac{۴,۵}{۳\sqrt{۳}} = \frac{۱,۵}{\sqrt{۳}} = \frac{\sqrt{۳}\,۱,۵}{۳} = \frac{\sqrt{۳}}{۲}$

$\sin\alpha = \frac{\sqrt{۳}}{۲} \rightarrow \alpha = ۶۰^\circ, ۱۲۰^\circ \quad \frac{۱۲۰}{۶۰} = \boxed{۲}$

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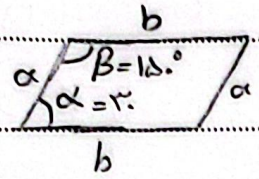
(۷)



$S = \sin 150^\circ \cdot ab = 54$

$\rightarrow S = \frac{1}{2} ab = 54 \rightarrow ab = 108$

$\frac{a}{b} = \frac{3}{2} \rightarrow a = \frac{3}{2} b$

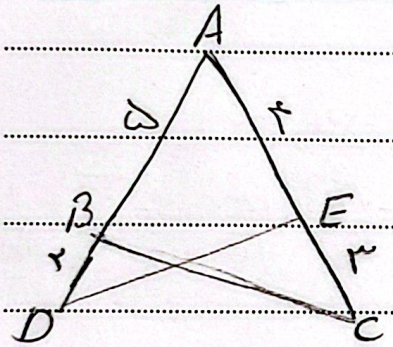
$\begin{cases} ab = 108 \\ a = \frac{3}{2} b \end{cases} \rightarrow \frac{3}{2} b \times b = 108 \rightarrow b^2 = 144 \rightarrow b = 9\sqrt{2}$

(اندازه مثبت هست)

$a = 6\sqrt{2}$

$P = 2(a+b) = 2(6\sqrt{2} + 9\sqrt{2}) = 30\sqrt{2}$

(۸) $\tan A$ ؟



$S_{ABC} - S_{ADE} = 1.75$

$\rightarrow \frac{1}{2} \times 5 \times 7 \sin \hat{A} - \frac{1}{2} \times 7 \times 4 \sin \hat{A} = 1.75$

$\rightarrow 17.5 \sin A - 14 \sin A = 1.75$

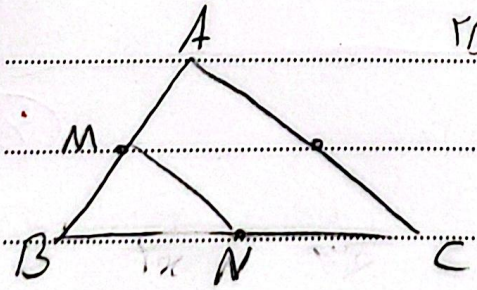
$\rightarrow 3.5 \sin \hat{A} = 1.75$

$\rightarrow \sin \hat{A} = \frac{1}{2} \rightarrow \hat{A} \in \{30^\circ, 150^\circ\}$ (غ ق ق)

$\rightarrow \sin^2 A + \cos^2 A = 1 \rightarrow \cos A = \frac{\sqrt{3}}{2}$ $\quad 1 + \tan^2 A = \frac{1}{\frac{3}{4}}$

$\rightarrow \tan A = \frac{\sqrt{3}}{3}$

(۹)



$2BN = 2NC \rightarrow NC = \frac{3}{2} BN$

$\frac{1}{2} AB \cdot BC \cdot \sin \hat{B} = 3 \times \frac{1}{2} \times BM \cdot BN \cdot \sin \hat{B}$

$AB \times BC = 3\, BM \times BN$

$(BM + AM) \times (BN + NC) = 3 \times BM \times BN$

$NC = \frac{3}{2} BN$

PARAMOUNT

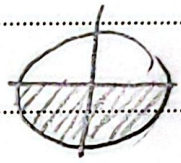
$\rightarrow \frac{3}{2}(BM+AM) = 3BM \rightarrow \frac{3}{2}BM + \frac{3}{2}AM = 3BM$

$\rightarrow \frac{3}{2}AM = \frac{3}{2}BM \rightarrow 3AM = 3BM \rightarrow \frac{BM}{AM} = \frac{3}{3}$

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} \rightarrow \frac{|\sin\alpha|}{\cos\alpha} = -\frac{\sin}{\cos}$ (۱۰)

$\frac{\cos}{\sin}$

$\rightarrow |\sin\alpha| = -\sin\alpha \rightarrow \sin < 0$ (۱)

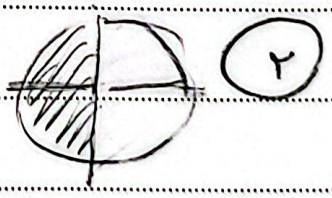


$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|}$

$\rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1+\sin\alpha}{|\cos\alpha|}$

$\rightarrow \frac{1}{|\cos\alpha|} - \frac{1+\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \rightarrow \frac{-\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha}$

$\rightarrow \cos\alpha < 0$



(۲)

(۱) ∩ (۲) $\rightarrow$