Date : Subject :

۱۵

$a_2 = \sqrt{a_3}$ → $a^2 q^2 = a q^3$ → $a q^2 = q^3$ → $q = 1$ → $q = \frac{1}{a}$

$a_5 = a q^4 = 27$ → $a \times \frac{1}{a^4} = a^{-3} = 27$ → $a = \frac{1}{3}$, $q = \frac{1}{3}$

$\frac{1}{4} - \frac{1}{3^3} = \frac{1}{6}$

تالیف ۱۸ — ۲۵ — برنام خدا — لیلا رمضانی هرپس B

الف) $\tan \frac{11\pi}{4} + \sin \frac{15\pi}{4} \cos \frac{13\pi}{4}$ (۱)

→ $\tan(2\pi + \frac{3\pi}{4}) + \sin(2\pi + \frac{3\pi}{4}) \cos(2\pi + \frac{\pi}{4})$

→ $\tan 135^\circ + \sin 315^\circ \times \cos 225^\circ = -1 + (-\frac{\sqrt{2}}{2} \times -\frac{\sqrt{2}}{2}) = -\frac{1}{2}$ ✓

ب) $\tan \frac{17\pi}{6} \sin \frac{11\pi}{3} + \cos \frac{10\pi}{3}$ (۲)

→ $\tan(2\pi + \frac{5\pi}{6}) \sin(2\pi + \frac{5\pi}{3}) + \cos(2\pi + \frac{\pi}{3})$

→ $\tan(150^\circ) \times \sin 300^\circ + \cos 240^\circ = -\frac{\sqrt{3}}{3} \times -\frac{\sqrt{3}}{2} + -\frac{1}{2} = \frac{1}{2}$ ✓

$\cot x = 4$ → $\frac{\cos x}{\sin x} = 4$ → $\cos x = 4 \sin x$ (۲)

$\frac{2\cos x - \sin x}{\cos x + \sin x} \xrightarrow{\cos x = 4\sin x} \frac{12\sin x - \sin x}{4\sin x + \sin x} = \frac{11\sin x}{5\sin x} = \frac{11}{5}$ ✓ (۲)

$\sin \alpha = 2\cos\alpha$ — ۳ ناحیه α (۳)

→ $\sqrt{1 - \cos^2\alpha} = 2\cos\alpha$ → $1 - \cos^2\alpha = 4\cos^2\alpha$ → $\cos^2\alpha = \frac{1}{5}$

→ $\cos\alpha = \pm\frac{\sqrt{5}}{5}$ (α ناحیه ۳) → $\cos\alpha = -\frac{\sqrt{5}}{5}$ ✓ (۲)

Date : Subject :

۴) α در ربع دوم

الف) $\sin\alpha = \frac{\sqrt{۲}}{۱۰}$ $\quad \sin^۲\alpha + \cos^۲\alpha = ۱ \rightarrow \frac{۲}{۱۰۰} + \cos^۲\alpha = ۱ \rightarrow \cos\alpha = -\frac{\sqrt{۹۸}}{۱۰}$

$\cos\left(\frac{۱۱\pi}{۴} + \alpha\right) = \cos\left(۱۳۵^\circ + \alpha\right) = \cos ۱۳۵^\circ \times \cos\alpha - \sin ۱۳۵^\circ \times \sin\alpha$

$= -\frac{\sqrt{۲}}{۲} \times \frac{۷\sqrt{۲}}{۱۰} - \frac{\sqrt{۲}}{۲} \times \frac{\sqrt{۲}}{۱۰} = \frac{۷}{۱۰} - \frac{۱}{۱۰} = \frac{۶}{۱۰} = \boxed{\frac{۳}{۵}}$ ✓

ب) α در ربع اول

$\tan\alpha = \frac{۱}{۷} \rightarrow \tan^۲\alpha + ۱ = \frac{۱}{\cos^۲\alpha} \rightarrow \frac{۵۰}{۴۹} = \frac{۱}{\cos^۲\alpha} \rightarrow \cos\alpha = \frac{۷}{\sqrt{۵۰}}$

$\tan\alpha = \frac{\sin\alpha}{\cos} = \frac{۱}{۷} \rightarrow \frac{\sin\alpha}{\frac{۷}{\sqrt{۵۰}}} = \frac{۱}{۷} \rightarrow \sin\alpha = \frac{۱}{\sqrt{۵۰}}$

۲

$\sin\left(\frac{۱۳\pi}{۴} + \alpha\right) = \sin\left(\left(۳\pi + \frac{\pi}{۴}\right) + \alpha\right) = \sin\left(۲۲۵^\circ + \alpha\right)$

$= \sin ۱۳۵^\circ \times \cos\alpha + \cos ۱۳۵^\circ \times \sin\alpha = \frac{-\sqrt{۲}}{۲} \times \frac{۷}{\sqrt{۵۰}} + \frac{-\sqrt{۲}}{۲} \times \frac{۱}{\sqrt{۵۰}}$

$= \frac{-۷}{۱۰} - \frac{۱}{۱۰} = -\frac{۸}{۱۰} = \boxed{-\frac{۴}{۵}}$ ✓

ج) $۲\sin^۲ x + \cos^۲ x = \frac{۴}{۳} \rightarrow \sin^۲ x + \sin^۲ x + \cos^۲ x = \frac{۴}{۳}$

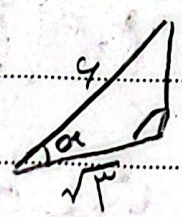
$\rightarrow \sin^۲ x = \frac{۱}{۳} \quad \rightarrow \sin^۲ x + \cos^۲ x = ۱ \rightarrow \frac{۱}{۳} + \cos^۲ x = ۱$

$\rightarrow \cos^۲ x = \frac{۲}{۳} \quad \tan^۲ x + ۱ = \frac{۱}{\cos^۲ x} \rightarrow \tan^۲ x = \frac{۱}{\frac{۲}{۳}} - ۱$

$\rightarrow \tan^۲ x = \boxed{\frac{۱}{۲}}$ ✓

۲

۴) $\frac{۱}{۲}\sin\alpha \sqrt{۲} \times ۶ = ۴,۵$

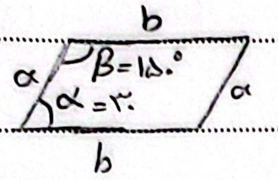


$\rightarrow \sin\alpha = \frac{۴,۵}{۳\sqrt{۲}} = \frac{۱,۵}{\sqrt{۲}} = \frac{\sqrt{۲}\,۱,۵}{۲} = \frac{\sqrt{۳}}{۲}$

$\sin\alpha = \frac{\sqrt{۳}}{۲} \rightarrow \alpha = ۶۰^\circ, ۱۲۰^\circ \quad \frac{۱۲۰}{۶۰} = \boxed{۲}$ ✓

PARAMOUNT

Date :   Subject :



۷

$S=\sin 150°\, ab = 54$

$\rightarrow S=\frac{1}{2}ab=54 \rightarrow ab=108$

$\frac{a}{b}=\frac{2}{3} \rightarrow a=\frac{2}{3}b$

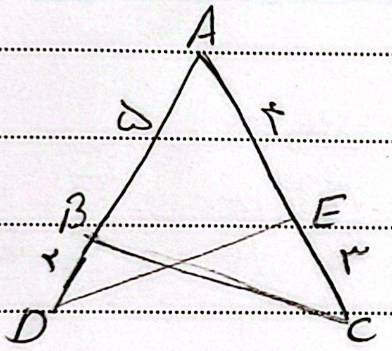
۲

$\begin{cases} ab=108 \\ a=\frac{2}{3}b \end{cases} \rightarrow \frac{2}{3}b\times b=108 \rightarrow b^2=\sqrt{162} \rightarrow b=9\sqrt{2}$

به اندازه مثبت هست

$a=6\sqrt{2}$

محیط $=2(a+b)=2(6\sqrt{2}+9\sqrt{2})=\boxed{30\sqrt{2}}$ ✓



? tan A ۸

$S_{\triangle ABC}-S_{\triangle ADE}=1.75$

$\rightarrow \frac{1}{2}\times 5\times 7\sin\hat{A}-\frac{1}{2}\times 7\times 4\sin\hat{A}=1.75$

$\rightarrow 17.5\sin A-14\sin A=1.75$

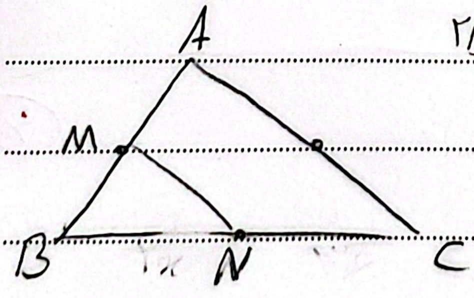
$\rightarrow 3.5\sin\hat{A}=1.75$

۲

$\rightarrow \sin\hat{A}=\frac{1}{2} \rightarrow \hat{A}\in\{30,\ 150\}$

$\rightarrow \sin^2 A+\cos^2 A=1 \rightarrow \cos A=\frac{\sqrt{3}}{2}$ $1+\tan^2 A=\frac{1}{\frac{\sqrt{3}}{2}}$

$\rightarrow \tan A=\boxed{\frac{\sqrt{3}}{3}}$ ✓



۹

$2BN=3NC \rightarrow NC=\frac{2}{3}BN$

$\frac{1}{2}AB.BC.\sin\hat{B}=3\times\frac{1}{2}\times BM.BN.\sin B$

$AB\times BC=3\,BM\times BN$

$(BM+AM)\times\underbrace{(BN+NC)}_{\frac{5}{3}BN}=3\times BM\times BN$

۲

Date : Subject :

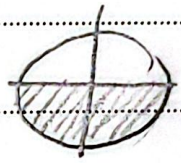
$\rightarrow \frac{3}{2}(BM+AM) = 2BM \rightarrow \frac{3}{2}BM + \frac{3}{2}AM = 2BM$

$\rightarrow \frac{3}{2}AM = \frac{1}{2}BM \rightarrow 3AM = BM \rightarrow \frac{BM}{AM} = \frac{3}{1}$ ✓

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} \rightarrow \frac{|\sin\alpha|}{\cos\alpha} = -\frac{\sin}{\cos}$ (۱۰)

$\frac{\cos}{\sin}$

$\rightarrow |\sin\alpha| = -\sin\alpha \rightarrow \sin < 0$ (۱)

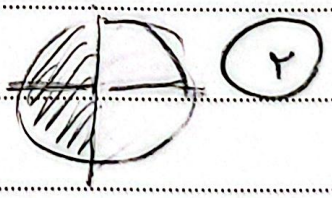


$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|}$

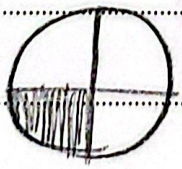
$\rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1+\sin\alpha}{|\cos\alpha|}$ (۲)

$\rightarrow \frac{1}{|\cos\alpha|} - \frac{1+\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \rightarrow \frac{-\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha}$

$\rightarrow \cos\alpha < 0$



(۲)

(۱) ∩ (۲) $\rightarrow$  ✓