(۱)

الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4}\cos\frac{13\pi}{4}$

$\tan\left(\frac{8\pi}{4}+\frac{3\pi}{4}\right) + \sin\left(\frac{12\pi}{4}+\frac{3\pi}{4}\right)\cos\left(\frac{12\pi}{4}+\frac{\pi}{4}\right)$

$\Rightarrow \tan 135 + \sin 135 \times \cos 45 \to -1 + \frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2} \to -1 + \frac{1}{2} = \boxed{\frac{-1}{2}}$

ب) $\tan\frac{17\pi}{4}\sin\frac{11\pi}{3} + \cos\frac{15\pi}{3}$

$\tan\left(\frac{3\pi}{4}+\frac{5\pi}{4}\right)\sin\left(\frac{4\pi}{3}+\frac{3\pi}{3}\right)+\cos\left(\frac{4\pi}{2}+\frac{4\pi}{2}\right)$

$-\frac{\sqrt{2}}{2}\times\left(\frac{\sqrt{2}}{2}\right)+1 = \boxed{0}$

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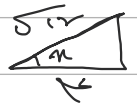
(۲)

$\cot\alpha = 4$



$\sin\alpha = \frac{1}{\sqrt{17}}$

$\cos\alpha = \frac{4}{\sqrt{17}}$

$\frac{3\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}$

$\frac{\frac{12}{\sqrt{17}}-\frac{1}{\sqrt{17}}}{\frac{4}{\sqrt{17}}+\frac{1}{\sqrt{17}}} = \frac{\frac{11}{\sqrt{17}}}{\frac{5}{\sqrt{17}}} = \frac{11\sqrt{17}}{5\sqrt{17}} = \boxed{\frac{11}{5}}$

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(۳)

$\sin^2\alpha + \cos^2\alpha = 1 \xrightarrow{\sin=2\cos} (2\cos\alpha)^2 + \cos^2\alpha = 1$

$5\cos^2\alpha = 1$

$\cos\alpha = -\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \boxed{-\frac{\sqrt{5}}{5}}$

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(۴)

الف) $\cos\left(\frac{11\pi}{2}+\alpha\right) \to \frac{11\pi}{2} = 2\pi + \frac{3\pi}{2} \to \cos\left(\frac{3\pi}{2}+\alpha\right)$

$\to \sin\alpha = \frac{\sqrt{7}}{10}$

$\cos\alpha = \sqrt{1-\sin^2\alpha} = -\frac{3\sqrt{7}}{10} \to \cos\left(\frac{3\pi}{2}+\alpha\right) = \cos\frac{3\pi}{2}\cos\alpha - \sin\frac{3\pi}{2}\sin\alpha \to$

$\to \left(-\frac{\sqrt{7}}{4}\right)\left(-\frac{3\sqrt{7}}{10}\right) - \left(\frac{\sqrt{7}}{4}\right)\left(\frac{\sqrt{7}}{10}\right) = \boxed{\frac{3}{5}}$

ب) $\tan^2\alpha + 1 = \frac{1}{\cos^2\alpha} \to 1 + \frac{1}{9} = \frac{1}{\cos^2\alpha} = \cos\alpha = \frac{3}{\sqrt{10}} = \frac{3\sqrt{7}}{10}$

$\sin\alpha = \frac{\sqrt{7}}{10}$

$\sin\left(\frac{5\pi}{2}+\alpha\right) = \left(-\frac{\sqrt{7}}{4}\right)\cos\alpha + \left(-\frac{\sqrt{7}}{4}\right)\sin\alpha = -\frac{\sqrt{7}}{4}(\sin\alpha+\cos\alpha) = \boxed{-\frac{4}{5}}$

(۵)

$\sin^2\alpha = 1-\cos^2\alpha \longrightarrow 2(1-\cos^2\alpha)+\cos^2\alpha = \frac{4}{3}$

$2-\cos^2\alpha = \frac{4}{3} \longrightarrow \cos^2\alpha = 2-\frac{4}{3} = \frac{2}{3}$ $\quad \sin^2\alpha = 1-\frac{2}{3} = \frac{1}{3}$

$\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

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(۶)

$S_{ABC} = \frac{1}{2} \times 9 \times 5\sqrt{3} \times \sin\alpha = 45$

$\frac{45}{5\sqrt{3}} = \frac{9}{5\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} = \sin\alpha$

زاویه مثلث $\longrightarrow 0 < \alpha < \pi$ $\quad \alpha = \frac{\pi}{3}\ (60^\circ)$ $\quad \alpha = \frac{2\pi}{3}\ (120^\circ)$

$\sin\alpha = \frac{\sqrt{3}}{2}$ $\quad \sin\alpha = \frac{\sqrt{3}}{2}$

$\frac{10\sqrt{3}}{60} = \frac{\sqrt{3}}{6}$ بدون تغییر

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(۷)

$a = 2R$ $\quad b = 3R$ $\quad S = ab\sin\alpha \longrightarrow$

$(2R)(3R)\sin 45 = 6R^2 \times \frac{1}{2} = 3R^2$

$3R^2 = 54 \longrightarrow R^2 = 18$ $\quad R \to 3\sqrt{2}$ $\quad$ اضلاع $\begin{cases} 2R = 6\sqrt{2} \\ 3R = 9\sqrt{2} \end{cases}$

$P = 2(6\sqrt{2}+9\sqrt{2}) = 2(15\sqrt{2}) = 30\sqrt{2}$

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(۸)

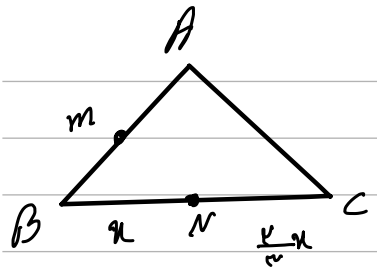
$AB = 5$ $\quad AD = 7$

$BD = 3$

$15\sin\alpha - 12\sin\alpha = 17.5$ $\quad 3\sin\alpha = 17.5$

$\sin\alpha = \frac{17.5}{35} = \frac{1}{2} \longrightarrow \hat{A} = 30^\circ \rightarrow \tan 30^\circ = \sqrt{3}/3$

(9)



$$S_{AB\hat{C}} = 4 \times S_{bmn}$$

$$4 \times \frac{1}{2} \times BM \times BN \times \sin\hat{B} = \frac{1}{2} \times AB \times BC \times \sin\hat{B}$$

$$4 \times BM = (BM + AM) \times 3 \longrightarrow 4BM = 3BM + 3AM$$

$$\longrightarrow BM = 3AM$$

$$BM = \frac{3}{1} AM \longrightarrow \frac{BM}{AM} = \frac{3}{1}$$

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(10)

① $\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \longrightarrow -\tan\alpha = \frac{1+\sin\alpha-1}{|\cos\alpha|} = \frac{\sin\alpha}{|\cos\alpha|}$

② $\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} \longrightarrow \frac{|\sin\alpha|}{\cos\alpha} = -\tan\alpha$

① ¿pelo → $\frac{-\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|}$ $\times \frac{1}{\sin\alpha}$ → $-\frac{1}{\cos\alpha} = \frac{1}{|\cos\alpha|} \longrightarrow |\cos\alpha| = -\cos\alpha \rightarrow \cos\alpha < 0$

② ¿pelo → $\frac{|\sin\alpha|}{\cos\alpha} = \frac{-\sin\alpha}{\cos\alpha} \longrightarrow |\sin\alpha| = -\sin\alpha$

$\sin\alpha < 0$

$\sin\alpha < 0$, $\cos\alpha < 0$ → cuadrante