محمدرضا حمیدی روش

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الف) $\tan \frac{11\pi}{4} + \sin \frac{15\pi}{4} \cos \frac{13\pi}{4}$

$\tan\left(\frac{8\pi}{4}+\frac{3\pi}{4}\right) + \sin\left(\frac{12\pi}{4}+\frac{3\pi}{4}\right)\cos\left(\frac{12\pi}{4}+\frac{\pi}{4}\right)$

$\Rightarrow \tan 135 + \sin 135 \times \cos 45 \rightarrow -1 + \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \rightarrow -1 + \frac{1}{2} = \boxed{-\frac{1}{2}}$ ✓

ب) $\tan \frac{17\pi}{6} \sin \frac{11\pi}{3} + \cos \frac{15\pi}{3}$

$\tan\left(\frac{12\pi}{6}+\frac{5\pi}{6}\right)\sin\left(\frac{6\pi}{3}+\frac{5\pi}{3}\right)+\cos\left(\frac{9\pi}{3}+\frac{6\pi}{3}\right)$

$-\sqrt{3}/3 \times (\sqrt{3}/2) + 1/2 = \boxed{0}$ ✓

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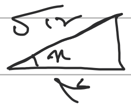
$\cot x = 4$



$\sin x = \frac{1}{\sqrt{17}}$

$\cos x = \frac{4}{\sqrt{17}}$

$\frac{3\cos x - \sin x}{\cos x + \sin x}$

$\frac{\frac{12}{\sqrt{17}} - \frac{1}{\sqrt{17}}}{\frac{4}{\sqrt{17}} + \frac{1}{\sqrt{17}}} = \frac{\frac{11}{\sqrt{17}}}{\frac{5}{\sqrt{17}}} = \frac{11\sqrt{17}}{5\sqrt{17}} = \boxed{\frac{11}{5}}$ ✓

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$\sin^2 + \cos^2 = 1 \xrightarrow{\sin = 2\cos} (2\cos\alpha)^2 + \cos^2 = 1$

$5\cos^2 = 1$

$\cos\alpha = -\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \boxed{-\frac{\sqrt{5}}{5}}$ ✓

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الف) $\cos\left(\frac{11\pi}{4}+\alpha\right) \rightarrow \frac{11\pi}{4} = 2\pi + \frac{3\pi}{4} \rightarrow \cos\left(\frac{3\pi}{4}+\alpha\right)$

$\rightarrow \sin\alpha = \frac{\sqrt{2}}{10}$

$\cos\alpha = 1 - \sin^2 = -\frac{7\sqrt{2}}{10} \rightarrow \cos\left(\frac{3\pi}{4}+\alpha\right) = \cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha \rightarrow$

$\rightarrow \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{-7\sqrt{2}}{10}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{10}\right) = \boxed{\frac{3}{5}}$ ✓

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ب) $\tan^2 + 1 = \frac{1}{\cos^2} \rightarrow 1 + \frac{1}{49} = \frac{1}{\cos^2\alpha} = \cos\alpha = \frac{7}{\sqrt{50}} = \frac{7\sqrt{2}}{10}$

$\sin\alpha = \frac{\sqrt{2}}{10}$

$\sin\left(\frac{5\pi}{4}+\alpha\right) = \left(-\frac{\sqrt{2}}{2}\right)\cos\alpha + \left(-\frac{\sqrt{2}}{2}\right)\sin\alpha = -\frac{\sqrt{2}}{2}(\sin\alpha + \cos\alpha) = \boxed{-\frac{4}{5}}$ ✓

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$\sin^2\alpha = 1-\cos^2\alpha \longrightarrow 2(1-\cos^2) + \cos^2\alpha = \frac{4}{3}$

$2-\cos^2 = \frac{4}{3} \longrightarrow \cos^2\alpha = 2-\frac{4}{3} = \frac{2}{3}$ $\quad \sin^2 = 1-\frac{2}{3} = \frac{1}{3}$

$\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ ✓

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$S_{ABC} = \frac{1}{2} \times 9 \times 5\sqrt{3} \times \sin\alpha = 45$

$\frac{45}{5\sqrt{3}} = \frac{9}{5\sqrt{3}} = \frac{3}{\sqrt{3}} = \sin\alpha$

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زاویه مثلث $\longrightarrow 0 < \alpha < \pi$ $\quad \alpha = \frac{\pi}{3}\ (60^\circ)$ $\quad \alpha = \frac{2\pi}{3}\ (120^\circ)$

$\sin\alpha = \frac{\sqrt{3}}{2}$ $\quad \sin\alpha = \frac{\sqrt{3}}{2}$

$\frac{120}{60} = 2$ بنابراین ✓

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$a = 2x$ $\quad b = 3x$ $\quad S = ab\sin\alpha \longrightarrow$

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$(2x)(3x)\sin 60 = 6x^2 \times \frac{1}{2} = 3x^2$

$3x^2 = 75 \longrightarrow x^2 = 25$ $\quad x \to 3\sqrt{5}$ $\quad$ اضلاع $\begin{cases} 2x = 6\sqrt{5} \\ 3x = 9\sqrt{5} \end{cases}$

$P = 2(6\sqrt{5} + 9\sqrt{5}) = 2(15\sqrt{5}) = 30\sqrt{5}$ ✓

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$AB = 5$ $\quad AD = 7$

$BD = 3$

$15\sin\alpha - 12\sin\alpha = 15\sqrt{3}$ $\quad 3\sin\alpha = 15\sqrt{3}$

$\sin\alpha = \frac{15\sqrt{3}}{3\sqrt{5}} = \frac{\sqrt{3}}{2} \longrightarrow \hat{A} = 30^\circ \rightarrow \tan 30^\circ = \frac{\sqrt{3}}{3}$ ✓

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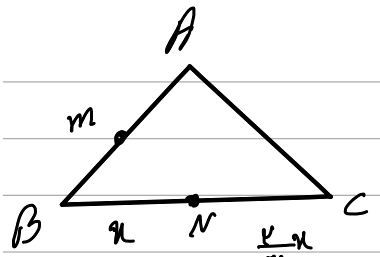

$S_{\overset{\Delta}{ABC}} = ٣ \times S_{\overset{\Delta}{bmn}}$

$٣ \times \frac{١}{٢} \times \overline{BM} \times \overline{BN} \times \sin B = \frac{١}{٢} \times \overline{AB} \times \overline{BC} \times \sin \hat{B}$

$٣ \times BM = (BM + AM) \times \frac{٥}{٣} \longrightarrow ٩BM = ٥BM + ٥AM$

$\longrightarrow ٤BM = ٥AM$

$BM = \frac{٥}{٤} AM \longrightarrow \frac{BM}{AM} = \frac{٥}{٤}$ ✓

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① $\frac{١}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{١+\sin\alpha}{|\cos\alpha|} \longrightarrow -\tan\alpha = \frac{١+\sin\alpha-١}{|\cos\alpha|} = \frac{\sin\alpha}{|\cos\alpha|}$

② $\frac{|\sin\alpha|}{\cos\alpha} = -\frac{١}{\cot\alpha} \longrightarrow \frac{|\sin\alpha|}{\cos\alpha} = -\tan\alpha$

① طبق $\longrightarrow \frac{-\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \xrightarrow{\alpha \; \frac{١}{\sin\alpha}} -\frac{١}{\cos\alpha} = \frac{١}{|\cos\alpha|} \longrightarrow |\cos\alpha| = -\cos\alpha \longrightarrow \cos\alpha < ٠$

② طبق $\longrightarrow \frac{|\sin\alpha|}{\cos\alpha} = \frac{-\sin\alpha}{\cos\alpha} \longrightarrow |\sin\alpha| = -\sin\alpha$

$\sin\alpha < ٠$

$\left.\begin{matrix}\sin\alpha < ٠ \\ \cos\alpha < ٠\end{matrix}\right\} \leadsto$ ربع سوم ✓

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