نام و نام خانوادگی ...[handwritten name]... پاسخنامه تشریحی تکلیف شماره ..۱۷.. کلاس ..دهم ریاضی..

۱	عدد آخر هر دسته = n^2 عدد اول هر دسته = $(n-1)^2+1$ $\{1\},\{2,3,4\},\{5,6,7,8,9\},\dots$ عدد آخر دسته ۹ : $9^2=81$ عدد اول : $8^2+1=65$ $\frac{81+65}{2}=73$
۲	دسته اول، دسته دوم، دسته سوم، دسته چهارم، دسته پنجم $\{1,2,3\},\{4,5,6,\dots,12\},\{13,14,15,\dots,39\},\{40,41,\dots,120\},\{121,122,\dots,363\}$ $\frac{121+363}{2}=242$
۳	if $n=3k \Rightarrow a_n=2^k \Rightarrow a_0=1,\ a_3=2,\ a_6=4,\ a_9=8$ $n=3k+1 \Rightarrow a_n=-2k+4 \Rightarrow a_1=4,\ a_4=2,\ a_7=0$ $n=3k+2 \Rightarrow a_n=[\frac{n}{k+2}]+a \Rightarrow a_2=[\frac{2}{2}]+a=-1$ $a_5=[\frac{5}{3}]+a=-1$ $a_8=[\frac{8}{4}]+a=0$ $1+2+4+8+4+2+0+1+a+1+a+2+a=19 \Rightarrow 3a+25=19 \Rightarrow a=-2$ $a_n=[\frac{n}{k+2}]-2 \xrightarrow{n=3k+2} a_n=[\frac{3k+2}{k+2}]-2=[2+\frac{k-2}{k+2}]-2=[\frac{k-2}{k+2}] \Rightarrow a_n=-1,1,0,0,0,\dots$ $\Rightarrow a_2+a_5+a_8+\dots+a_{29}=-2$
۴	$a_n=an^2+bn+c$ $a_5=14=25a+5b+c$ $a_7=17/2=49a+7b+c$ $\Rightarrow 24a+2b=3/2 \Rightarrow 12a+b=1/6$ $\Rightarrow -2/4+b=1/6 \Rightarrow b=4$ $a=\frac{1}{70}\times(-14)=-\frac{1}{5}$ $a_5=25\times-\frac{1}{5}+5\times4+c=14 \Rightarrow c=-1$ $a_{15}=225\times-\frac{1}{5}+60-1=14$ $a_1=-\frac{1}{5}+4-1=2/8$ $\frac{a_{15}}{a_1}=\frac{14}{2/8}=5$
۵	$a_4=b_2$, $a_8=b_7$ $\Rightarrow a_1+3d=b_2$, $a_1+7d=b_7$ $\Rightarrow 4d=b_7-b_2=7m+b-2m-b=5m$ $\Rightarrow 5m=4d$ $b_n=am+b$ $b_{10}=0 \Rightarrow 10m+b=0 \Rightarrow b=-10m$ $d=\frac{b_{15}}{b_1} \Rightarrow b_1=\frac{b_{15}}{d}=\frac{15m-10m}{\frac{5}{4}m}=\frac{5m}{\frac{5}{4}m}=4$

$6a_2^2 = 5a_3 a + 3a_2 a \Rightarrow 6(a+d)^2 = 5(a+2d)a + 3(a+d)a$

$\Rightarrow 6a^2 + 12ad + 6d^2 = 5a^2 + 10ad + 3a^2 + 3ad$ 6

$\Rightarrow -2a^2 - ad + 6d^2 = 0 \xrightarrow{\text{روی}} a^2 - ad - 12d^2 = 0 \Rightarrow (a-4d)(a+3d) = 0 \Rightarrow a \begin{cases} \frac{4d}{-2} = -2d \\ \frac{-3d}{-2} = 1/5d \end{cases}$

$\frac{a_4}{d} = \frac{a_1 + 3d}{d} = \begin{cases} \frac{-2d + 3d}{d} = 1 \\ \frac{1/5d + 3d}{d} = 4/5 \end{cases}$

$b = \frac{a+c}{2} \quad (\frac{a}{2})^2 = \frac{bc}{4} \Rightarrow a^2 = bc \Rightarrow a^2 = (\frac{a+c}{2}) \times c \Rightarrow a^2 = \frac{ac + c^2}{2} \Rightarrow 2a^2 - ac - c^2 = 0$

$\xrightarrow{\text{روی}} a^2 - ac - 2c^2 = 0 \Rightarrow (a-2c)(a+c) = 0$ 7

$\Rightarrow a = \begin{cases} \frac{2c}{2} = c \rightarrow \text{غیرثابت} \times \\ \frac{-c}{2} \checkmark \end{cases}$

$a_n = b, \frac{a}{2}, \frac{c}{4} \Rightarrow a_n = b, -\frac{c}{4}, \frac{c}{4}$ ($q = -1$)

$\Rightarrow q = -1 \Rightarrow 2q = -2$

$2a = \frac{3b + c}{2} \quad b^2 = ac \Rightarrow b^2 = \frac{3bc + c^2}{4} \Rightarrow 4b^2 - 3bc - c^2 = 0 \xrightarrow{\text{روی}} b^2 - 3bc - 4c^2 = 0$

$\Rightarrow (b + c)(b - 4c) = 0 \Rightarrow b \begin{cases} \frac{-c}{4} \checkmark \\ c \times \end{cases} \quad q = \frac{c}{b} = \frac{c}{\frac{-c}{4}} = -4$ 8

$\frac{t_9}{t_6} = q^3 = (-4)^3 = -64$

$\frac{aq^5}{a^3q^3} + \frac{aq}{a^2} = 2 \Rightarrow \frac{q^2}{a^2} + \frac{aq}{a^2} = 2 \Rightarrow 2a^2 - q^2 - aq = 0$

$\xrightarrow{\text{روی}} a^2 - aq - 2q^2 = 0 \Rightarrow (a - 2q)(a + q) = 0 \Rightarrow a \begin{cases} a = \frac{2q}{2} = q \\ a = \frac{-q}{2} \end{cases}$ 9

$\frac{a^2}{aq} = \frac{a}{q} = \begin{cases} \frac{q}{q} = 1 \\ \frac{\frac{-q}{2}}{q} = -\frac{1}{2} \end{cases}$

$\left. \begin{array}{l} a_3 = \sqrt{a_4} \\ a_5 = 27 \end{array} \right\} \Rightarrow a_5 \times a_3 = a_4^2 \Rightarrow 27\sqrt{a_4} = \sqrt{a_4^4} \Rightarrow 27 = \sqrt{a_4^3}$

$\Rightarrow \sqrt{3^6} = \sqrt{a_4^3} \Rightarrow a_4 = 3^2 = 9 \quad q = \frac{a_5}{a_4} = \frac{27}{9} = 3$ 10

$a_4 = aq^3 \Rightarrow 9 = 27a_1 = a_1 = \frac{1}{3} \quad \frac{1}{2} - a_1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$