

$$\{1\}, \{2, 3, 4\}, \{5, 6, 7, 8, 9\}, \dots$$

$$n^2 = \text{عدد آخر مرتبه}$$

$$(n-1)^2 + 1 = \text{عدد اول مرتبه}$$

$$a^2 = 1 \Rightarrow a = 1$$

$$a^2 + 1 = 4 \Rightarrow a = 2$$

$$\frac{1+4}{2} = 2.5$$

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$$\{1, 2, 3\}, \{4, 5, 6, 7, 8, 9\}, \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}, \dots$$

$$\frac{1+4+9+16}{4} = 7.5$$

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$$n = 3k \Rightarrow a_n = 2k \Rightarrow a_0 = 1, a_3 = 2, a_6 = 4, a_9 = 1$$

$$n = 3k+1 \Rightarrow a_n = -2k \Rightarrow a_1 = 2, a_4 = 4, a_7 = 0$$

$$n = 3k+2 \Rightarrow a_n = \left[\frac{n}{3}\right] + a = \left[\frac{2}{3}\right] + a = -1$$

$$a_n = \left[\frac{n}{3}\right] + a = -1$$

$$a_n = \left[\frac{n}{3}\right] - 1 \Rightarrow a_n = \left[\frac{2k+2}{3}\right] - 1 = 0$$

$$a_n = \left[\frac{n-2}{3}\right] \Rightarrow a_n = 1, 0, 0, 1, 0, 0, \dots$$

$$a_n = a_n^2 + b_n + c \Rightarrow \begin{cases} a_0 = 1 = 2a_0 + b_0 + c \\ a_1 = 2 = 2a_1 + b_1 + c \\ a_2 = -1 = 2a_2 + b_2 + c \end{cases} \Rightarrow \begin{cases} 2a + b + c = 1 \\ 2a + b + c = 2 \\ -2a + b + c = -1 \end{cases}$$

$$a = \frac{1}{3} \times (-1) = -\frac{1}{3}$$

$$a_0 = 2a_0 + b_0 + c = 1 \Rightarrow c = -1$$

$$a_n = b_n \Rightarrow a_1 = b_1 = 2, a_2 = b_2 = -1, a_3 = b_3 = 4, \dots$$

$$d = \frac{b_1}{b_0} = \frac{2}{1} = 2$$

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$$y_a^r = \omega_a a + r_a a \Rightarrow y(a, d)^r = \omega(a, r, d) a + r(a, d) a$$

$$\Rightarrow y_a^r + r_a d, y_d^r = \omega_a^r + 1 \cdot a d + r_a^r, r_a d$$

$$\Rightarrow -r_a^r - a d + y_d^r = 0 \quad \text{--- (53)} \quad a^r - a d - r_d^r = 0 \Rightarrow (a - r_d)(a + r_d) = 0 \Rightarrow a = \begin{cases} \frac{r_d}{-r} = -r_d \\ \frac{-r_d}{-r} = 1/r_d \end{cases}$$

$$\frac{a}{d} = \frac{a + r_d}{d} = \begin{cases} -\frac{r_d r_d}{d} = -1 \\ 1/\omega_d + r_d = r/\omega \end{cases}$$

$$b = \frac{a+c}{r} \quad \left(\frac{a}{r}\right)^r = \frac{b}{r} \Rightarrow a^r = b/r \Rightarrow a^r = \left(\frac{a+c}{r}\right) \times c \Rightarrow d^r = \frac{ac+c^r}{r} = r_a^r - ac - c^r = 0$$

$$a_n = b, \frac{a}{r}, \frac{c}{r} \Rightarrow a_n = b, -\frac{c}{r}, \frac{c}{r}$$

$$\Rightarrow q = -1 \Rightarrow r_a = (-r) \quad \checkmark$$

$$a^r - ac - r_c^r = 0 \Rightarrow (a - r_c)(a + r_c) = 0 \Rightarrow a = \begin{cases} \frac{r_c}{r} = c \\ -\frac{c}{r} \end{cases}$$

$$r_a = \frac{r b + c}{r} \quad b^r = ac \Rightarrow b^r = \frac{r b c + c^r}{r} \Rightarrow r b^r - r b c - c^r = 0 \quad \text{--- (54)} \quad b^r - r b c - r_c^r = 0$$

$$\Rightarrow (b + c)(b - r_c) = 0 \Rightarrow b = \begin{cases} -\frac{c}{r} \\ c \end{cases} \quad \checkmark \quad q = \frac{c}{b} = \frac{c}{-\frac{c}{r}} = -r$$

$$\frac{t_a}{t_g} = q^r = (-r)^r = (-r^r) \quad \checkmark$$

$$\frac{a q^{\omega}}{a^r q^r} + \frac{a q}{a^r} = r \Rightarrow \frac{q^r}{a^r} + \frac{a q}{a^r} = r \Rightarrow r a^r - a^r - a q = 0$$

$$\text{--- (55)} \quad a^r - r a q - q^r = 0 \Rightarrow (a - r_q)(a + q) = 0 \Rightarrow a = \begin{cases} \frac{r_q}{r} = q \\ -\frac{q}{r} \end{cases}$$

$$\frac{a^r}{a q} = \frac{a}{q} = \begin{cases} \frac{a}{q} = 1 \\ -\frac{a}{r} = -\frac{1}{r} \end{cases} \quad \checkmark$$

$$\left. \begin{array}{l} a_r = \sqrt{a_f} \\ a_{\omega} = r_v \end{array} \right\} \Rightarrow a_{\omega} a_r = a_f^r \Rightarrow r_v \sqrt{a_r} = \sqrt{a_f^r} \Rightarrow r_v = \sqrt{\frac{a_f^r}{a_r}}$$

$$\Rightarrow \sqrt{r_v^2} = \sqrt{\frac{a_f^r}{a_r}} \Rightarrow a_f = r_v^2 = q \quad q = \frac{a_{\omega}}{a_f} = \frac{r_v}{q} = r \quad \checkmark$$

$$a_f = a q^r \Rightarrow q = r_v a_1 = \left(\frac{a_1}{r} = \frac{1}{r} \right) \quad \frac{1}{r} - a_1 = \frac{1}{r} - \frac{1}{r} = \left(\frac{1}{r} \right) \quad \checkmark$$