

$$\{\dots, 1^2\} \dots \{9^2\} \rightarrow \{\omega, \dots, 6^2\} \dots \{1^2\}$$

۸ دسته ۹ دسته ۸ دسته ۹ دسته

$$\frac{11+6\omega}{2} = \boxed{73}$$

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$$\{1, 2, 3\} \dots \{4, \dots, 12\} \dots \{13, \dots, 21\} \dots \{22, \dots, 30\} \dots \{31, \dots, 36\}$$

۱ دسته ۲ دسته ۳ دسته ۴ دسته ۵ دسته

$$\frac{363+121}{2} = \boxed{242}$$

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$$\frac{4}{1} = \frac{1+a}{2} = \frac{2}{3} = \frac{2}{4} = \frac{1+a}{5} = \frac{4}{6} = \frac{0}{7} = \frac{2+a}{8} = \frac{1}{9} = \frac{-2}{10}$$

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$$S_6 = 2 + 1 + a + 2 + 2 + 1 + a + 4 + 0 + 2 + a + 1 - 2 = 19 \Rightarrow 22 + 2a = 19 \Rightarrow 2a = -3 \Rightarrow a = -1$$

$$\frac{0}{2} = \frac{0}{4} = \frac{1}{6} = \frac{1}{8} = \frac{1}{10} = \frac{1}{12} = \frac{1}{14} = \frac{1}{16} = \frac{1}{18} = \frac{1}{20} \rightarrow 0 + 0 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = \boxed{10}$$

$$x n^2 + y n + z$$

$$x = \frac{1}{V_0} \times (-14) = -0.12$$

$$\left. \begin{aligned} n=0 & \rightarrow -0.12 \times 2\omega + \omega + z = 14 \Rightarrow -\omega + \omega + z = 14 \\ n=1 & \rightarrow -0.12 \times 49 + V + z = 14/2 \Rightarrow -9/1 + V + z = 14/2 \end{aligned} \right\} \Rightarrow -4/1 + 2V = 2/2$$

$$\Rightarrow y = 1 \Rightarrow y = 2 \Rightarrow -\omega + \omega(2) + z = 14 \Rightarrow -\omega + 2\omega + z = 14 \Rightarrow \omega + z = 14 \Rightarrow z = -1$$

$$\rightarrow a_n = -0.12n^2 + 2n - 1 \Rightarrow \frac{a_n}{a_1} = \frac{-0.12n^2 + 2n - 1}{-0.12 + 2 - 1} = \frac{12n^2 - 24n + 12}{1} = 12(n-1)^2 = \boxed{12}$$

$$\left. \begin{aligned} a_r &= a'_r \Rightarrow a + rd = a' + d' \\ a_1 &= a'_1 \Rightarrow a + d = a' + d' \end{aligned} \right\} \Rightarrow rd = \omega d' \Rightarrow d' = \frac{rd}{\omega}$$

$$a'_1 = a' + 9d' = 0 \Rightarrow a' = -9d'$$

$$\frac{a'_1}{d} = \frac{a' + 14d'}{d} = \frac{-9d' + 14d'}{d} = \frac{5d'}{d} = \frac{\omega(\frac{rd}{\omega})}{d} = \frac{rd}{d} = \boxed{r}$$

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$$ar = a + d \rightarrow r(a+d)^r = \omega a(a+rd) + r a(a+d) \rightarrow r a^r + r d^r + 1 r a d = \omega a^r + 1 \omega a d + r a^r + r a d$$

$$\rightarrow r a^r + a d - r d^r = 0$$

$$a = \frac{-d \pm \sqrt{d^r - r(r)(-r d^r)}}{r} = \frac{-d \pm \sqrt{r d^r}}{r} = \frac{-d \pm \sqrt{r} d}{r} \rightarrow \begin{cases} a = -\sqrt{r} d \\ a = \frac{r d}{r} \end{cases}$$

$$a = -\sqrt{r} d \rightarrow \frac{a^r}{d} = \frac{d}{a} = 1$$

$$a = \frac{r d}{r} \rightarrow \frac{a^r}{d} = \frac{r d^r}{d} = \frac{r}{r}$$

$$a, b, c \rightarrow a, a + d, a + rd$$

$$b = \frac{a}{r}, c = \frac{a}{r} \rightarrow d + a = \frac{a}{r} \rightarrow \frac{a + rd}{r}$$

$$\frac{a^r}{r} = \frac{a^r + r d a + r d^r}{r} \Rightarrow a^r = a^r + r d a + r d^r \Rightarrow r d a + r d^r = 0 \Rightarrow r d a = -r d^r \Rightarrow r a = -r d$$

$$\Rightarrow a = -\frac{r d}{r} \Rightarrow d = -\frac{r a}{r}$$

$$r | -\frac{r a}{r} | = \boxed{r a}$$

$$a, b, c \rightarrow a, a q, a q^r \rightarrow r b, r a, c \rightarrow r a q, r a, a q^r$$

$$r b = a + c \Rightarrow r a = r a q + a q^r \Rightarrow r = r q + q^r \Rightarrow q^r + r a - r = 0 \Rightarrow q = 1 \text{ (not possible)} \Rightarrow q = r$$

$$\frac{a q}{a q^r} = \frac{a q^r}{a q^d} = q^r = r = \boxed{r}$$

$$q^r = -r$$

$$q = -r$$

$$\frac{a q^d}{a^r q^r} + \frac{a q}{a^r} = \frac{q^r}{a^r} + \frac{q}{a} = r$$

$$\frac{a q^r + a^r q}{a^r} = r \Rightarrow a q (q + a) = r a^r \Rightarrow q (q + a) = r a^r \Rightarrow q^r + a q = r a^r \Rightarrow a^r = \frac{q^r + a q}{r}$$

$$\Rightarrow a q = r a^r - a^r$$

$$\frac{a^r}{a q} = \frac{q^r + a q}{r a^r - a^r} = \frac{q^r + a q}{r a^r - a^r}$$

$$\sqrt{a^r} = a^r \Rightarrow \sqrt{a q^r} = a q^r \Rightarrow a q^r = a^r q^r \Rightarrow a q = 1 \Rightarrow a = \frac{1}{q}$$

$$a_d = r V \Rightarrow a q^r = r V \Rightarrow \frac{1}{q} \cdot q^r = r V \Rightarrow q^r = r V \Rightarrow q = r \Rightarrow a = \frac{1}{r}$$

$$\left| \frac{1}{r} - \frac{1}{r} \right| = \left| \frac{r}{r} - \frac{r}{r} \right| = \left| -\frac{1}{r} \right| = \boxed{\frac{1}{r}}$$

سوال ۳

$$\begin{cases} a_0 = 1 \\ a_2 = 2 \\ a_4 = 4 \\ a_6 = 8 \end{cases} \quad \begin{cases} a_1 = 4 \\ a_3 = 2 \\ a_5 = 0 \end{cases}$$

$$\begin{cases} a_2 = a+1 \\ a_3 = a+1 \\ a_4 = a+2 \end{cases}$$

میانگین اول $\leftarrow a_0, a_2, a_4, a_6$
میانگین دوم $\leftarrow a_1, a_3, a_5$
میانگین سوم $\leftarrow a_4, a_5, a_6$

$$S_1 = 1 + 4 + (a+1) + 2 + 2 + (a+1) + 4 + 0 + (a+2) + 8 = 14 \rightarrow a = -2$$

جمع جملات $a_2 + a_3 + \dots$ از میانگین سوم:

$$a_n = \left\lfloor \frac{n}{k+2} \right\rfloor + a \xrightarrow{n=3k+2} \left\lfloor \frac{3k+2}{k+2} \right\rfloor - 2 \rightarrow a_n = \left\lfloor \frac{2(k+2)}{k+2} + \frac{k-2}{k+2} \right\rfloor - 2$$

$$\rightarrow S_n = \left\lfloor 2 + \frac{k-2}{k+2} \right\rfloor - 2 = \frac{k-2}{k+2}$$

معادله $\left\lfloor \frac{k-2}{k+2} \right\rfloor$ به ازای $k=0, k=1$ برابر 1- است، به ازای سایر اعداد حسابی برابر صفر است:

$$\sum_{k=0}^4 \left\lfloor \frac{k-2}{k+2} \right\rfloor = -2$$

سوال ۷

$$a, b, c \xrightarrow{\text{حسابی}} b = \frac{a+c}{2} \rightarrow a = 2b - c$$

$$b, \frac{a}{r}, \frac{c}{r} \xrightarrow{\text{هندسی}} \frac{a}{r} = b \times \frac{c}{r} \rightarrow a^r = bc$$

$$(2b-c)^r = bc \rightarrow 2^r b^r - 2^r b^r c + c^r = 0$$

$$(b-c)(2^r b^r - c^r) = 0 \rightarrow \begin{cases} b=c \\ 2^r b^r = c^r \end{cases} \xrightarrow{\text{جملات متناهی}} \begin{cases} b=c \\ 2b=c \end{cases}$$

$$\text{دنباله هندسی} \rightarrow b, \frac{a}{r}, \frac{c}{r} \rightarrow b, \frac{2b}{r}, \frac{2b}{r}$$

$$2q = -2$$

$$q = -1 \leftarrow b, -b, b$$

$$a = -2b \leftarrow \frac{a+2b}{2} = b$$

سوال ۹

$$\frac{a_4}{(a_2)^2} + \frac{a_2}{(a)^2} = \frac{aq^4}{(aq)^2} + \frac{aq}{a^2} = \frac{aq^2}{a^2q^2} + \frac{aq}{a^2} = 2 \rightarrow \frac{q^2}{a^2} + \frac{q}{a} = 2$$

$$\frac{q}{a} = t \rightarrow t^2 + t - 2 = 0 \rightarrow \begin{cases} t = -2 \rightarrow \frac{q}{a} = -2 \\ t = 1 \rightarrow \frac{q}{a} = 1 \end{cases}$$

$$\frac{a^r}{a_r} = \frac{a^r}{aq} = \frac{a}{q} \rightarrow \begin{cases} \frac{a}{q} = -\frac{1}{2} \\ \frac{a}{q} = 1 \end{cases}$$