

$$\{1\}, \{2, 3, 4\}, \{5, 6, 7, 8, 9\}, \dots$$

$$n = \text{شماره است}$$

$$9^2 = 81 \Rightarrow \text{عدد اول دسته کف} = n^2 \Rightarrow \text{عدد اول دسته بعدی} = n^2 + 1$$

$$\Rightarrow \left. \begin{aligned} 81 + 1 = 82 \\ \text{عدد اول دسته کف} = 81 \\ \text{عدد اول دسته بعدی} = 82 \end{aligned} \right\} \Rightarrow \text{میانگین اعداد} = \frac{81 + 82}{2} = 81.5$$

$$\{1, 2, 3\}, \{4, 5, 6, \dots, 12\}, \{13, 14, 15, \dots, 39\}, \dots$$

$$\text{تعداد اعداد در دسته} = 3^3 = 27 \Rightarrow \text{تعداد اعداد در دسته بعدی} = 3^4 = 81$$

$$3 + 9 + 27 + \dots + 81 = 3 \times \frac{3^4 - 1}{3 - 1} = 120$$

$$\Rightarrow \text{میانگین اعداد} = \frac{120 + 81}{2} = 100.5$$

$$S_0 = a_0 + a_1 + a_2 + \dots + a_n = 19$$

$$n = 2k \Rightarrow n = 0, 2, 4, 6, \dots \Rightarrow \begin{cases} a_0 = 2^0 = 1 \\ a_2 = 2^2 = 4 \\ a_4 = 2^4 = 16 \\ a_6 = 2^6 = 64 \end{cases}$$

$$n = 2k+1 \Rightarrow n = 1, 3, 5, \dots \Rightarrow \begin{cases} a_1 = 2^1 = 2 \\ a_3 = 2^3 = 8 \\ a_5 = 2^5 = 32 \end{cases}$$

$$n = 2k+2 \Rightarrow n = 2, 4, 6, \dots \Rightarrow \begin{cases} a_2 = 2^2 = 4 \\ a_4 = 2^4 = 16 \\ a_6 = 2^6 = 64 \end{cases}$$

$$\left. \begin{aligned} n = 2 &\Rightarrow 2k+2 = 2 \Rightarrow k = 0 \\ n = 4 &\Rightarrow 2k+2 = 4 \Rightarrow k = 1 \\ n = 6 &\Rightarrow 2k+2 = 6 \Rightarrow k = 2 \end{aligned} \right\} \Rightarrow \text{در این حالت مجموع اعداد برابر است با}$$

$$10a + \left\{ \left[\frac{2}{2} \right] + \left[\frac{4}{2} \right] + \left[\frac{6}{2} \right] + \left[\frac{8}{2} \right] + \dots + \left[\frac{18}{2} \right] \right\} = 10a + \{1 + 2 + 3 + \dots + 9\} = 10a + 45 = 10(-2) + 45 = 25$$

$$an^2 + bn + c = 14 \Rightarrow \begin{cases} 4a + 2b + c = 14 \\ 9a + 7b + c = 17 \end{cases} \Rightarrow 5a + 5b = 3 \Rightarrow a + b = 0.6$$

$$a = \frac{1}{5} \times (-14) = -\frac{14}{5} \Rightarrow 2 \times (-\frac{14}{5}) + 2b = 3 \Rightarrow 2b = 3 + \frac{28}{5} = \frac{43}{5} \Rightarrow b = \frac{43}{10}$$

$$\Rightarrow 4 \times (-\frac{14}{5}) + 5 \times (\frac{43}{10}) + c = 14 \Rightarrow c = -1 \Rightarrow \text{مجموع} = -\frac{1}{5}n^2 + \frac{43}{10}n - 1$$

$$\frac{a_1 a_2}{a_1} = \frac{-\frac{1}{5} \times 15^2 + \frac{43}{10} \times 15 - 1}{-\frac{1}{5} + \frac{43}{10} - 1} = \frac{-45 + 64.5 - 1}{-\frac{2}{10} + \frac{43}{10} - \frac{10}{10}} = \frac{18.5}{\frac{31}{10}} = \frac{185}{31} = 5$$

$$t_n = a + b \Rightarrow t_1 = 1 \cdot a + b = 0 \Rightarrow b = -1 \cdot a$$

$$b_1 = b + (n-1)d \Rightarrow b_1 + 4d = -1a \Rightarrow \left. \begin{array}{l} b_1 + 4d = -1a \\ b_1 + 4d = -3a \end{array} \right\} \Rightarrow \frac{b_1 + 4d}{b_1 + 4d} = \frac{-1a}{-3a} \Rightarrow$$

$$b_1 = -4d \Rightarrow b_1 = -\frac{4}{9}d = -9 \cdot \frac{1}{9}d \Rightarrow -9 \cdot \frac{1}{9}d + 4d = -1a \Rightarrow$$

$$\frac{t_{10}}{d} = \frac{10a}{d} = \frac{10(-1a)}{d} = -10 \Rightarrow \text{جواب اولی}$$

$$4(a+d)^2 = 5a(a+4d) + 3a(a+d) \Rightarrow -4a^2 - ad + 4d^2 = 0 \Rightarrow \Delta = d^2 - 4(-1)d = d^2 + 4d$$

$$\Rightarrow a = \frac{d \pm \sqrt{d^2 + 4d}}{-1} \Rightarrow \left. \begin{array}{l} a = -d \\ a = \frac{4}{9}d \end{array} \right\} \Rightarrow \frac{a}{d} = \frac{-d}{d} \Rightarrow \frac{a}{d} = -1 \Rightarrow \frac{a}{d} = \frac{4}{9} = \frac{4}{9}$$

$$a, b, c \Rightarrow 2b = a + c$$

$$b, \frac{a}{r}, \frac{c}{r} \Rightarrow \frac{a}{r} = \frac{b}{r} \Rightarrow ar = bc \Rightarrow$$

$$(2b-c)^2 = bc \Rightarrow 4b^2 - 4bc + c^2 = bc \Rightarrow 4b^2 - 5bc + c^2 = 0 \Rightarrow (b-c)(b-c) = 0 \Rightarrow b = c \Rightarrow b = \frac{c}{r} \Rightarrow b = \frac{c}{r}$$

$$2b = a + c \Rightarrow \left. \begin{array}{l} \text{اگر } b = c \Rightarrow 2c = a + c \Rightarrow a = c \\ \text{اگر } b = \frac{c}{r} \Rightarrow \frac{2c}{r} = a + c \Rightarrow a = -\frac{c}{r} \end{array} \right\} \Rightarrow q = \frac{\frac{c}{r}}{-\frac{c}{r}} = -1 \Rightarrow r_q = -1$$

$$a, b, c \Rightarrow b^2 = ac \Rightarrow a = \frac{b^2}{c}$$

$$2b, 2a, c \Rightarrow 2a = 2b + c$$

$$\Rightarrow \frac{4b^2}{c} = \frac{2b+c}{1} = \frac{c^2 + 2bc}{c} \Rightarrow$$

$$4b^2 - 2bc - c^2 = 0 \Rightarrow (b-c)(b+c) = 0 \Rightarrow b = c \Rightarrow b = \frac{c}{r} \Rightarrow b = \frac{c}{r}$$

$$a = \frac{b^2}{c} = \frac{\frac{c^2}{16}}{c} = \frac{c}{16}$$

$$\text{جواب دوم: } \frac{c}{16}, -\frac{c}{16}, c, \dots \Rightarrow q = -\frac{1}{16}$$

$$\frac{a_q}{a_q} = \frac{a_q \times q^r}{a_q} = (-1)^r = -1$$

$$\frac{a_q}{(a_q)^r} + \frac{a_q}{a^r} = 2 \Rightarrow \frac{a_q}{a^r} + \frac{a_q}{a^r} = 2 \Rightarrow \frac{a_q + a_q}{a^r} = 2 \Rightarrow$$

$$2a_q - a_q - q^r = 0 \Rightarrow a_q - q^r = (a - q)(a + q) \Rightarrow a = \frac{q^r}{q} = q$$

$$\text{اگر } a = q \Rightarrow \frac{a^r}{a_q} = \frac{q^r}{q^r} = 1 \Rightarrow \text{اگر } a = -\frac{q}{r} \Rightarrow \frac{a^r}{a_q} = \frac{(-\frac{q}{r})^r}{-\frac{q}{r}} = -\frac{1}{r}$$

$$\text{جواب سوم: } a_q = q^r \Rightarrow a_q^r = a^r q^r \Rightarrow a_q = 1$$

$$a_q^r = 2V \Rightarrow \frac{a_q^r}{a_q} = 2V \Rightarrow q^r = 2V \Rightarrow q = \sqrt[2]{2V} \Rightarrow a = \frac{1}{\sqrt[2]{2V}} \Rightarrow \text{جواب چهارم: } \frac{1}{\sqrt[2]{2V}}, 1, \sqrt[2]{2V}, \dots \Rightarrow q = \sqrt[2]{2V}$$