نام و نام خانوادگی: ایلیا ...... ۱۱ پاسخنامه تشریحی تکلیف شماره ۱۰ کلاس دهم B ......

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$\{1\}$ (1) $\{2,3,4\}$ (2) $\{5,6,7,8,9\}$ (3) , ... $\{(n-1)^2+1, \dots, n^2\}$ (n)

$n=9 \Rightarrow \{65, \dots, 81\}$ ⇒ را سطرهای $= \frac{65+81}{2} = 73$

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$\{1,2,3\}$ (1), $\{4,5,6,7,8,9,10,11,12\}$ (2), $\{13, \dots, 39\}$ (3), $\{40, \dots, 120\}$ (4), $\{121, \dots, 363\}$ (5)

میانگین ← دو سطر های بین اول و آخری ← $\frac{121+363}{2} = 242$

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$a_5 = 14 , a_7 = 17.2$

$14 , x , 17.2$

$a_n = \frac{1}{5}n^2 + bn + c$

$14 = 5 + 5b + c \Rightarrow 9 = 5b + c$

$17.2 = 9.8 + 7b + c \Rightarrow 7.4 = 7b + c$

$\Rightarrow b = -0.8 , c = 13$

$\Rightarrow \frac{a_{15}}{a_1} = \frac{\frac{15^2}{5} - (0.8 \times 15) + 13}{\frac{1}{5} - \frac{8}{10} + 13} = \frac{46}{12.4} = \frac{110}{31}$

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$a_4 = b_2 \Rightarrow a + 3d_1 = b + d_2$

$a_8 = b_7 \Rightarrow a + 7d_1 = b + 6d_2$

$\Rightarrow 10d_2 = 7d_2 \Rightarrow d_1 = \frac{7}{10}d_2$

$b_{10} = 0 \Rightarrow b = -9d_2$

$b_{15} = b + 14d_2 \Rightarrow b_{15} = 5d_2$

$\frac{b_{15}}{d_1} = \frac{5d_2}{\frac{7}{10}d_2} = \frac{50}{7}$

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۱) $9(a+d)^2 = 9a^2 + 18ad + 9a^2 + 9ad \Rightarrow 9d^2 = 9a^2 + 9ad$

$\frac{a_4}{d} = \frac{a+3d}{d} = \frac{a}{d} + 3 \Rightarrow \frac{a}{d} + 3 = t \Rightarrow t^2 + t = 9 \Rightarrow t^2 + t - 9 = 0$

$t = \frac{-1 \pm \sqrt{37}}{2} \begin{cases} \rightarrow t = \frac{5}{2} \checkmark \rightarrow \frac{a_4}{d} = \frac{a_1+3d}{d} = \frac{d}{d} = 1 \\ \rightarrow t = -3 \rightarrow \frac{a_4}{d} = \frac{a_1+3d}{d} = \frac{9d/2}{d} = 4.5 \end{cases}$

۲) $b-d, b, b+d \qquad b, \frac{b-d}{2}, \frac{b+d}{2} \qquad \frac{b^2+bd}{2} = \frac{b^2-2bd+d^2}{2} \Rightarrow d^2 - 2bd = 0$

$\Rightarrow d(d-2b) = 0 \Rightarrow d = 2b \qquad b, \frac{b-2b}{2}, \frac{b+2b}{2}$

$b, -b, b \Rightarrow q = -1, 2q = -2 \checkmark$

۳) $a, aq, aq^2 \qquad 2a, 2aq, aq^2 \Rightarrow q^2 + 2q - 2 = 0$

$2aq + aq^2 = 8 \Rightarrow 2q + q^2 = 4$

$q = \frac{-2 \pm \sqrt{12}}{2} \begin{cases} \rightarrow q = -4 \text{ ق ق} \\ \rightarrow q = 1 \text{ غ ق} \end{cases} \Rightarrow \frac{aq^5}{aq^3} = q^2 = (-4)^2 = 16 \checkmark$

۴) $\frac{aq^3}{a^2q^2} + \frac{aq}{a^2} = 2 \Rightarrow \frac{q}{a^2} + \frac{q}{a} = 2 \qquad \frac{a^2}{aq} = \frac{a}{q} \qquad \frac{q^2}{a^2} + \frac{q}{a} - 2 = 0$

$(\frac{q}{a})^2 + \frac{q}{a} - 2 = 0 \Rightarrow \frac{q}{a} = \frac{-1 \pm \sqrt{9}}{2} \begin{cases} \rightarrow \frac{q}{a} = 1 \\ \rightarrow \frac{q}{a} = -2 \end{cases} \Rightarrow \frac{a}{q} = \begin{cases} \rightarrow 1 \checkmark \\ \rightarrow -\frac{1}{2} \checkmark \end{cases}$

۵) $(aq^3)^2 = a^2q^6 \times aq^2 \Rightarrow a^2q^6 = a^3q^8 \Rightarrow aq^2 = q^2 \Rightarrow q^2 = 27 \Rightarrow q = \sqrt{27}$

$\Rightarrow aq^2 = 27 \Rightarrow 27a = 27 \Rightarrow a = \frac{1}{27}$

$\frac{25}{27} - \frac{5}{27} = \frac{20}{27}$

$a_3 = \sqrt{a_4} \rightarrow aq^2 = \sqrt{aq^3} \rightarrow a^2q^4 = aq^3 \rightarrow aq = 1$

$a_5 = 27 \rightarrow aq^4 = 27 \rightarrow (aq)q^3 = 27 \rightarrow q = 3$

$aq = 1 \xrightarrow{q=3} a = \frac{1}{3}$

$\left|\frac{1}{4} - a\right| = \left|\frac{1}{4} - \frac{1}{3}\right| = \boxed{\frac{1}{12}}$

$$\begin{cases} a_0 = 1 \\ a_2 = 2 \\ a_4 = 4 \\ a_6 = 8 \end{cases} \quad \begin{cases} a_1 = 4 \\ a_3 = 2 \\ a_5 = 0 \end{cases}$$

سوال ۳

ضابطه اول ← a_9, a_4, a_2, a_0

ضابطه دوم ← a_5, a_3, a_1

ضابطه سوم ← a_8, a_6, a_2

$$\begin{cases} a_2 = a+1 \\ a_6 = a+1 \\ a_8 = a+2 \end{cases}$$

$$S_{10} = 1+4+(a+1)+2+2+(a+1)+4+0+(a+2)+8 = 19 \rightarrow a = -2$$

جمع جملات $a_2 + a_6 + \dots$ از ضابطه سوم:

$$a_n = \left[\frac{n}{k+2}\right] + a \xrightarrow{n = 3k+2} \left[\frac{3k+2}{k+2}\right] - 2 \rightarrow a_n = \left[\frac{2(k+2)}{k+2} + \frac{k-2}{k+2}\right] - 2$$

$$\rightarrow S_n = \left[2 + \frac{k-2}{k+2}\right] - 2 = \frac{k-2}{k+2}$$

عبارت $\left[\frac{k-2}{k+2}\right]$ به ازای $k=0$ و $k=1$ برابر -1 است و به ازای سایر اعداد حسابی برابر صفر است:

$$\sum_{k=0}^{4} \left[\frac{k-2}{k+2}\right] = -2$$

سوال ۴

$$t_n = an^2 + bn + c \rightarrow \begin{cases} t_5 = 25a + 5b + c = 14 \\ t_7 = 49a + 7b + c = 19,2 \end{cases} \rightarrow \begin{array}{l} 24a + 2b = 5,2 \\ 12a + b = 2,6 \end{array}$$

$$a = \frac{1}{10}(-t_5) = \frac{1}{10}(-14) = -\frac{7}{5} \rightarrow a = \frac{-7}{5}$$

$$b = 4 \quad , \quad c = -1$$

$$t_{15} = -\frac{7}{5}(15)^2 + 4(15) - 1 = 14$$

$$t_1 = -\frac{7}{5}(1)^2 + 4(1) - 1 = 2,8$$

$$\frac{t_{15}}{t_1} = \frac{14}{2,8} = 5$$

سوال ۵

$$\begin{array}{l} a_4 = b_2 \rightarrow a_1 + 3d = b_2 \rightarrow 4d = b_7 - b_2 \\ a_8 = b_7 \rightarrow a_1 + 7d = b_7 \\ b_n = c_n + k \rightarrow b_7 - b_2 = (7c+k) - (2c+k) = 5c \end{array} \Big\} \rightarrow 5c = 4d$$

$$b_{10} = 0 \rightarrow 10c + k = 0 \rightarrow k = -10c$$

$$\frac{b_{15}}{d} = \frac{15c + k}{d} \rightarrow \frac{15c - 10c}{d} = \frac{5c}{d} = \frac{4d}{d} \rightarrow \frac{b_{15}}{d} = 4$$