

... و $\{2, 3, 4\}$ و $\{1\}$

$$\text{مجموعه آخر} = n^2 \Rightarrow \text{مجموعه آخر} = 9^2 = 81$$

$$\text{واسطه حسابی} \Rightarrow \frac{81 + 1}{2} = 41$$

$$\text{مجموعه اول} = \text{مجموعه آخر} + 1 = (n-1)^2 + 1 = (9-1)^2 + 1 = 64 + 1 = 65$$

... و $\{12, 13, \dots, 14\}$ و $\{2, 3, 4\}$ و $\{1\}$

$$\text{مجموعه اول} = 1, 4, 9, 16, 25 \Rightarrow \text{مجموعه آخر} = 12, 13, 14 \Rightarrow 12 \times 3 = 36 \Rightarrow$$

$$\text{میانگین اعداد} = \frac{121 + 363}{2} = 242$$

$$a_0, a_3, a_6, a_9 \rightarrow \text{ضابطه اول} \quad \left\{ \begin{aligned} \left[\frac{n}{k+2} \right] - 2 &\xrightarrow{n=3k+2} \left[\frac{3k+2}{k+2} \right] - 2 = \end{aligned} \right.$$

$$a_1, a_4, a_7 \rightarrow \text{ضابطه دوم}$$

$$a_2, a_5, a_8 \rightarrow \text{ضابطه سوم}$$

$$\left[2 + \frac{k-2}{k+2} \right] - 2 = \left[\frac{k-2}{k+2} \right] \rightarrow \text{به ازای هر } k \text{ برابر است و به ازای بقیه اعداد صفر است پس جواب ۲}$$

$$S_{10} = 1 + 4(1+a) + 2 + (4a) + 4 + 0 + (2+a) + 1 = 19 \Rightarrow a = -2$$

$$a_n = an^2 + bn + c$$

$$\left. \begin{aligned} 25a + 5b + c &= 14 \\ 49a + 7b + c &= 17.2 \end{aligned} \right\} \xrightarrow{\text{از هم کم کنیم}} 24a + 2b = 3.2 \Rightarrow b = 4 \Rightarrow c = -1 \Rightarrow$$

$$a = \frac{1}{70} \times (-14) = -0.2$$

$$\left. \begin{aligned} a_1 &= -0.2 + 4 - 1 = 3.8 \\ a_{15} &= -0.2(225) + (4 \times 15) - 1 = 14 \end{aligned} \right\} \Rightarrow \frac{14}{3.8} = 3.68 \text{ برابر}$$

$$\left. \begin{aligned} a_4 &= b_2 \\ a_1 &= b_5 \end{aligned} \right\} \Rightarrow \left. \begin{aligned} a_1 + 3d &= b_1 + d \\ a_1 + 4d &= b_1 + 4d \end{aligned} \right\} \Rightarrow$$

$$3d = d \Rightarrow d = \frac{4}{3}d \quad (*)$$

$$\left. \begin{aligned} b_{10} &= 0 \Rightarrow b_1 + 9d = 0 \Rightarrow b_1 = -\frac{9}{10}d \\ b_1 + \frac{36}{5}d &= 0 \Rightarrow b_1 = -\frac{36}{5}d \end{aligned} \right\} \Rightarrow$$

$$\frac{b_{10}}{d} = \frac{-\frac{36}{5}d + \frac{9}{10}d}{d} = \left(\frac{20}{5} \right) \Rightarrow \text{جواب ۴}$$

$$f(a_1+d) = a_1(a_1+d) + a_1(a_1+d) \Rightarrow$$

$$5a_1^2 + 10a_1d + 5d^2 = 5a_1^2 + 10a_1d + 5a_1^2 + 5a_1d \rightarrow 5a_1^2 + 10a_1d - 5d^2 = 0 \Rightarrow$$

$$a = \frac{-d \pm \sqrt{d^2 + 4d^2}}{5} = \frac{-d \pm \sqrt{5}d}{5} = \begin{cases} a = \frac{\sqrt{5}d}{5} \Rightarrow \frac{a}{d} = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}} \\ a = -\frac{4d}{5} \Rightarrow \frac{a}{d} = -\frac{4}{5} \end{cases}$$

$$a_n: a \cdot b \cdot c$$

$$b_n: b \cdot \frac{a}{r} \cdot \frac{c}{r} \xrightarrow{\text{مساواة}} c \cdot b \cdot r \leq b \cdot \frac{b-d}{r} \cdot \frac{b+d}{r} \Rightarrow$$

$$\frac{1}{r} (b-d)^2 = \frac{b}{r} (b+d) \Rightarrow d^2 - 4bd = 0 \Rightarrow d \neq 0 \Rightarrow d = 4b$$

$$a_n: b \cdot b \cdot b \Rightarrow b_n = b \cdot b \cdot b \Rightarrow q = -1 \Rightarrow r = -1$$

$$a \cdot b \cdot c \Rightarrow b = aq, c = aq^2$$

$$r \cdot b \cdot c \xrightarrow{\text{مساواة}} r \cdot aq \cdot aq^2 \Rightarrow r - 4aq = aq^2 - a \Rightarrow$$

$$aq^2 + 4aq - r = 0 \Rightarrow a(q^2 + 4q - r) = 0 \xrightarrow{a \neq 0} (q+1)(q-1) = 0 \Rightarrow q = -1 \text{ or } q = 1$$

$$\frac{aq}{ar} = \frac{aq^2}{ar^2} = q = -1$$

$$\frac{aq^2}{ar^2} + \frac{aq}{ar} = r \Rightarrow \left(\frac{q^2}{a^2}\right) + \frac{q}{a} - r = 0 \Rightarrow \frac{q}{a} = u \Rightarrow u^2 + u - r = 0 \Rightarrow (u+1)(u-1) = 0 \Rightarrow$$

$$\frac{q}{a} = 1 \Rightarrow \frac{a}{aq} = 1, \frac{q}{a} = -1 \Rightarrow \frac{a}{q} = -1$$

$$a_r = aq^r$$

$$a_f = aq^f = (aq^r)^f \Rightarrow aq^f = a^f q^{rf} \Rightarrow aq^f = 1$$

$$a_d = aq^f = rV$$

$$\frac{1}{r} - \frac{1}{r} = \frac{1}{r} \Rightarrow aq = 1 \Rightarrow a = \frac{1}{r}$$

$$\frac{aq^f}{ar} = \frac{rV}{1} = r \Rightarrow q^f = rV = r^2 \Rightarrow q = r$$