نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$\{1\}\quad \{2,3,4\}\quad \{5,6,7,8,9\}\quad \{(n-1)^2+1,\dots,n^2\}$ ← n

$n=9 \leftarrow \{45,\dots,81\} \leftarrow$ واسطه $= \frac{45+81}{2} = 63$

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$\{1,2,3\}, \{4,5,6,7,8,9,10,11,12\}, \{13,\dots,39\}, \{40,41,\dots,120\}$

$, \{121,\dots,363\}$

$\frac{121+363}{2} = 242$

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$a_5 = 14$

$a_7 = 17/2$

$a_n = \frac{1}{8}n^2 + bn + c$

$14 = 5 + 5b + c \Rightarrow 9 = 5b + c$

$17/2 = 9/8 + 7b + c \Rightarrow 7/4 = 7b + c$

$\Downarrow$

$b = -0/08 \qquad c = 14$

$\frac{a_{10}}{a_1} = \frac{\frac{15^2}{8} - (0/08 \times 15) + 14}{\frac{225}{80} - \frac{8}{15} + 14} = \frac{440}{124} = \frac{110}{31}$

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$a_4 = b_2 \Rightarrow a + 3d_1 = b + d_2$

$a_8 = b_7 \Rightarrow a + 7d_1 = b + 9d_2$

$10d_2 = 7d_2 \Rightarrow d_1 = \frac{7}{10}d_2$

$b_{10} \Rightarrow b = -9d_2$

$b_{15} = b + 14d_2 \Rightarrow b_{15} = 5d_2$

$\frac{b_{15}}{d_1} = \frac{5d_2}{\frac{7}{10}d_2} = \frac{50}{7}$

$4(a+d)^2 = 9a^2 + 10ad + 3a^2 + 2ad \Rightarrow 9d^2 = 12a^2 + 12ad$

$\frac{a^2}{a}, \frac{a+2d}{d} = \frac{a}{d} + 2 \Rightarrow \frac{a}{d} + 2 = t \Rightarrow 2t^2 + t = 4a^2t^2 + t - 4 = 0$

$t = \frac{-1 \pm \sqrt{33}}{4}$ (1)

$t = \frac{4}{4}$ → $a = -2d$ → $\frac{a_4}{d} = \frac{a_1 + 3d}{d} = \frac{d}{d} = 1$

$t = -2$ → $a = \frac{3d}{2}$ → $\frac{a_4}{d} = \frac{a_1 + 3d}{d} = \frac{\frac{9d}{2}}{d} = \frac{9}{2}$

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$b-d, b, b+d$ $\quad b, \frac{b-d}{2}, \frac{b+d}{2}$ $\quad \frac{b+bd}{4} = \frac{b^2 - 2db + d^2}{4} \Rightarrow$

$\Rightarrow d(d-2b) \Rightarrow d = 2b$

$d^2 - 2bd = 0$

$b, -b, b \Rightarrow a = 1, 2a = -2$ ✓ ✓

$b, \frac{b-2b}{2}, \frac{b+2b}{2}$

(2)

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a, aq, aq^2 $\quad 4aq, 2a, aq^2$ $\}$ $a^2 + 2q - 6 = 0$

$4aq + aq^2 = 8a \Rightarrow 2a + q^2 = 8$

$q = \frac{-2 \pm \sqrt{28}}{2}$ $\quad q = -4$ ✓ $\quad q = 1$ ✗

$\Rightarrow \frac{aq^7}{aq^5} \Rightarrow q^2 = (-4)^2 = 16$ ✓

(2)

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$\frac{aq^6}{a^2q^2} + \frac{aq}{a^2} = 2 \Rightarrow \frac{q^4}{a^2} = \frac{q}{a} = 2$ $\quad \frac{a^2}{aq} = \frac{a}{q}$ $\quad \frac{q^2}{a^2} = \frac{q}{a} - 2 = 0$

(2)

$(\frac{q}{a})^2 + \frac{q}{a} - 2 = 0 \Rightarrow \frac{q}{a} = \frac{-1 \pm \sqrt{9}}{2}$ $\quad \frac{q}{a} = 1$ $\quad \frac{q}{a} = -2$ $\Rightarrow \frac{a}{q} < \frac{1}{-\frac{1}{2}}$ ✓

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$(aq^2)^2 = aq^4 \times aq^2 \Rightarrow a^2q^4 = aq^6 \times aq^2 \Rightarrow aq^6 = q^4 \Rightarrow q^2 = 27 \Rightarrow q = \sqrt{27}$

(5)

$\Rightarrow aq^6 = 27 \Rightarrow 729a = 27 \Rightarrow a = \frac{1}{27}$

$\frac{27}{54} - \frac{2}{54} = \frac{25}{54}$

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سوال ۳

$$\begin{cases} a_0 = 1 \\ a_2 = 2 \\ a_4 = 4 \\ a_6 = 8 \end{cases} \quad \begin{cases} a_1 = 4 \\ a_3 = 2 \\ a_5 = 0 \end{cases}$$

ضابطه اول ← a_6, a_4, a_2, a_0

ضابطه دوم ← a_5, a_3, a_1

ضابطه سوم ← a_8, a_6, a_4

$$\begin{cases} a_2 = a+1 \\ a_4 = a+1 \\ a_8 = a+2 \end{cases}$$

$$S_{10} = 1+4+(a+1)+2+2+(a+1)+4+0+(a+2)+8 = 19 \rightarrow a = -2$$

جمع جملات $a_2 + a_4 + \dots$ از ضابطه سوم:

$$a_n = \left[\frac{n}{k+2}\right] + a \xrightarrow{n = 2k+2} \left[\frac{2k+2}{k+2}\right] - 2 \rightarrow a_n = \left[\frac{2(k+2)}{k+2} + \frac{k-2}{k+2}\right] - 2$$

$$\rightarrow S_n = \left[2 + \frac{k-2}{k+2}\right] - 2 = \frac{k-2}{k+2}$$

عبارت $\left[\frac{k-2}{k+2}\right]$ به ازای $k=0$ و $k=1$ برابر -1 است و به ازای سایر اعداد حسابی برابر صفر است:

$$\sum_{k=0}^{4} \left[\frac{k-2}{k+2}\right] = -2$$

سوال ۴

$$t_n = an^2 + bn + c \rightarrow \begin{cases} t_5 = 25a + 5b + c = 14 \\ t_7 = 49a + 7b + c = 11,2 \end{cases} \rightarrow \begin{matrix} 24a + 2b = 3,2 \\ 12a + b = 1,6 \end{matrix}$$

$$a = \frac{1}{10}(-t_5) = \frac{1}{10}(-14) = -\frac{1}{5} \rightarrow a = \frac{-1}{5}$$

$$b = 4 \quad , \quad c = -1$$

$$t_{15} = -\frac{1}{5}(15)^2 + 4(15) - 1 = 14$$

$$t_1 = -\frac{1}{5}(1)^2 + 4(1) - 1 = 2,8$$

$$\frac{t_{15}}{t_1} = \frac{14}{2,8} = 5$$

سوال ۵

$$\begin{matrix} a_4 = b_2 \rightarrow a_1 + 3d = b_2 \rightarrow 4d = b_7 - b_2 \\ a_8 = b_7 \rightarrow a_1 + 7d = b_7 \\ b_n = c_n + k \rightarrow b_7 - b_2 = (7c+k) - (2c+k) = 5c \end{matrix} \Big\} \rightarrow 5c = 4d$$

$$b_{10} = 0 \rightarrow 10c + k = 0 \rightarrow k = -10c$$

$$\frac{b_{15}}{d} = \frac{15c + k}{d} \longrightarrow \frac{15c - 10c}{d} = \frac{5c}{d} = \frac{4d}{d} \rightarrow \boxed{\frac{b_{15}}{d} = 4}$$

سوال ۱۰

$$a_3 = \sqrt{a_6} \rightarrow aq^2 = \sqrt{aq^3} \rightarrow a^2q^4 = aq^3 \rightarrow aq = 1$$

$$a_5 = 27 \rightarrow aq^4 = 27 \rightarrow (aq)q^3 = 27 \rightarrow q = 3$$

$$aq = 1 \xrightarrow{q=3} a = \frac{1}{3}$$

$$\left|\frac{1}{2} - a\right| = \left|\frac{1}{2} - \frac{1}{3}\right| = \boxed{\frac{1}{6}}$$