نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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الف) $27^\circ = (x)\,rad \longrightarrow \frac{27}{180} = \frac{x}{\pi} \Rightarrow \frac{3}{20} = \frac{x}{\pi} \Rightarrow 3\pi = 20x \Rightarrow x = \frac{3\pi}{20} = 0/15\pi$

ب) $125^\circ = (x)\,rad \longrightarrow \frac{125}{180} = \frac{x}{\pi} \Rightarrow \frac{25}{36} = \frac{x}{\pi} \Rightarrow x = \frac{25\pi}{36}$

ج) $\frac{5\pi}{12}\,rad = (x)^\circ \longrightarrow \frac{\frac{5\pi}{12}}{\frac{\pi}{180}} = \frac{x}{180} \Rightarrow \frac{5}{1} = \frac{x}{180} \Rightarrow 12x = 180 \times 5 \Rightarrow x = \frac{180 \times 5}{12} = 75^\circ$

د) $\frac{4\pi}{9}\,rad = (x)^\circ \longrightarrow \frac{\frac{4\pi}{9}}{\frac{\pi}{180}} = \frac{x}{180} \Rightarrow \frac{4}{1} = \frac{x}{180} \Rightarrow x = 80^\circ$ (گزینه ۲) ✓

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$\frac{\frac{125}{9}a}{\frac{25}{36}} + \frac{\frac{5a\pi}{8}}{\frac{\pi}{1}} = \frac{5a}{72} + \frac{a}{8} + 10a = \frac{5a+9a}{72} + 10a = \frac{14a}{72} + 10a = \frac{7a}{36} + 10a$

$= \frac{367a}{36} \rightarrow \frac{367a}{36} = 180 \Rightarrow 367a = 180 \times 36 \Rightarrow a = \frac{180 \times 36}{367} \Rightarrow a = 17/45$

$\frac{D}{180} = \frac{rad}{\pi} \rightarrow \frac{a\pi}{\pi} = \frac{D_1}{180} \rightarrow D_1 = 22/5a$ دهم

$\frac{12/5a}{5} = D_2 \rightarrow D_2 = 12/5a$ دهم

$22/5a + 12/5a + 10a = 180$

$a = 4^\circ$

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الف) $\cos 40^\circ \cos 30^\circ - \sin 40^\circ \sin 30^\circ - \tan^2 45^\circ + 2\cot 45^\circ \overset{①,②}{\Longrightarrow} 0 + \tan^2 45^\circ = 1$

$\cos 40^\circ = \sin 50^\circ$, $\cos 30^\circ = \sin 60^\circ$ $\Rightarrow \cos 40^\circ \cos 30^\circ - \sin 40^\circ \sin 30^\circ = \sin 60^\circ \sin 40^\circ - \sin 40^\circ \sin 30^\circ = 0$ (۱)

$\sin 45^\circ = \cos 45^\circ \Rightarrow (\sin 45^\circ)^2 = (\cos 45^\circ)^2 \Rightarrow \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{\cos 45^\circ}{\sin 45^\circ} \Rightarrow \tan 45^\circ = \cot 45^\circ$

$\Rightarrow -\tan^2 45^\circ + 2\cot 45^\circ = -\tan^2 45^\circ + 2\tan 45^\circ = \tan 45^\circ$ (۲)

ب) $\frac{\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ}{\cot 30^\circ - \cot 60^\circ} = \frac{\frac{1}{3} + 2 + 1}{\sqrt{3} - \frac{1}{\sqrt{3}}} = \frac{\frac{10}{3}}{\frac{2}{\sqrt{3}}} = \frac{10\sqrt{3}}{6}$ (گزینه ۲) ✓

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$-\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = \sin^2\theta \Rightarrow \frac{1}{4} + \frac{3}{4} = \frac{1}{2} = \sin^2\theta \Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}$

θ حاده $\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$

$\cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$ (گزینه ۲) ✓

$\sin^2 + \cos^2 = 1 \Rightarrow \frac{1}{2} + \cos^2 = 1 \Rightarrow \cos^2 = \frac{1}{2} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$ θ حاده $\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

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$\tan 30^\circ = \cot 60^\circ \Rightarrow \tan^2 30^\circ = \cot^2 60^\circ$

$\Rightarrow \frac{2\tan 30^\circ(1 - \tan^2 30^\circ)}{(1 - \cot^2 60^\circ)^2} = \frac{2\tan 30^\circ(1 - \cot^2 60^\circ)}{(1 - \cot^2 60^\circ)^2} = \frac{2\tan 30^\circ}{1 - \cot^2 60^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$= \frac{3}{\sqrt{3}} = \sqrt{3} = \tan\theta$

$\Rightarrow \theta = 60^\circ, 240^\circ$ θ حاده $\Rightarrow \theta = 60^\circ$

$\frac{60^\circ}{180^\circ} = \frac{x}{\pi} \Rightarrow \frac{x}{\pi} = \frac{1}{3} \Rightarrow x = \frac{\pi}{3}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\cot^2 60^\circ = \frac{1}{3}$ (گزینه ۲) ✓

(راهنمای سوال ۳ قسمت «ب») $\tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow \tan^2 30^\circ = \frac{1}{3}$

$\cot 30^\circ = \frac{1}{\tan 30^\circ} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

$\tan 60^\circ = \sqrt{3} \Rightarrow \tan^2 60^\circ = 3$

$\cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}}$

$\tan 45^\circ = 1 \Rightarrow \tan^2 45^\circ = 1$

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$\tan\theta = 5 \Rightarrow \dfrac{\sin\theta}{\cos\theta} = 5 \Rightarrow \sin\theta = 5\cos\theta$

$\Rightarrow \dfrac{3\sin\theta - \cos\theta}{\sin\theta - 4\cos\theta} = \dfrac{15\cos\theta - \cos\theta}{5\cos\theta - 4\cos\theta} = \dfrac{14\cos\theta}{\cos\theta} = \boxed{14}$

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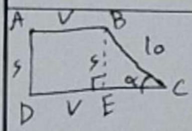
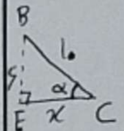


$\sin\alpha = \dfrac{\overline{BE}}{\overline{BC}} = \dfrac{6}{10}, \quad \overline{BE} = 6 \Rightarrow \overline{BC} = 10$



$6^2 + x^2 = 10^2 \Rightarrow 36 + x^2 = 100 \Rightarrow x^2 = 64 \Rightarrow x = \pm 8 \rightarrow$ چون طول است $\Rightarrow x = 8$

محیط ذوزنقه $= 6 + 7 + 10 + 8 + 7 = \boxed{38}$

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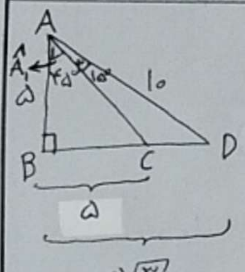

$\hat{A} = 60^\circ \Rightarrow \begin{cases} \sin\hat{A} = \dfrac{\sqrt{3}}{2} = \dfrac{\overline{BD}}{10} \Rightarrow \overline{BD} = 5\sqrt{3} \\ \cos\hat{A} = \dfrac{1}{2} = \dfrac{\overline{AB}}{10} \Rightarrow \overline{AB} = 5 \end{cases}$

$\hat{A}_1 = 45^\circ \Rightarrow \tan\hat{A}_1 = 1 = \dfrac{\overline{BC}}{5} \Rightarrow \overline{BC} = 5$

$\Rightarrow \overline{CD} = \overline{BD} - \overline{BC} = \boxed{5\sqrt{3} - 5 = 5(\sqrt{3} - 1)}$

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الف)  → ~~در ناحیه چهارم~~

در ناحیه سوم با افزایش θ، سینوس از ۰ به ۱- کاهش و کسینوس از ۱- به ۰ افزایش می‌یابد

ب)  → در ناحیه دوم

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$\tan x = \dfrac{1}{3} \Rightarrow \dfrac{\sin x}{\cos x} = \dfrac{1}{3} \Rightarrow 3\sin x = \cos x$

$x \rightarrow$ در ناحیه سوم $\Rightarrow \begin{cases} \sin < 0 \\ \cos < 0 \end{cases}$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + (3\sin x)^2 = 1 \Rightarrow \sin^2 x + 9\sin^2 x = 1 \Rightarrow 10\sin^2 x = 1 \Rightarrow \sin^2 x = \dfrac{1}{10}$

$\Rightarrow \sin x = \pm\dfrac{\sqrt{10}}{10} = \dfrac{\pm 1}{\sqrt{10}} \xrightarrow{\text{چون در ناحیه سوم است}} \boxed{\sin x = -\dfrac{\sqrt{10}}{10} = \dfrac{-1}{\sqrt{10}}}$

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