نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$a + aq + aq^2 = 21$

$a \cdot aq \cdot aq^2 = a^3q^3 = 64 \Rightarrow aq = 4 \Rightarrow a = \frac{4}{q}$

$\frac{4}{q} + 4 + 4q = 21 \Rightarrow \frac{4}{q} + 4q = 17 \overset{\times q}{\Rightarrow} 4q^2 - 17q + 4 = 0$

$q = \frac{17 \pm \sqrt{289-64}}{8} = \frac{17 \pm 15}{8} \Rightarrow \boxed{q = \frac{1}{4}}$ یا $q = 4 \rightarrow$ دوجمله‌ای

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$x^2+4, 2x, x^2-2$ تشکیل $(2x)^2 = (x^2+4)(x^2-2) \Rightarrow 4x^2 = x^4 + 2x^2 - 8$

$2x^2 = x^4 - 8 \Rightarrow x^4 - 2x^2 - 8 = 0 \Rightarrow t = x^2 \Rightarrow t^2 - 2t - 8 = 0 \Rightarrow \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2}$

$\Leftrightarrow t = 4 \Rightarrow x^2 = 4 \Rightarrow x = 2$ $\Leftrightarrow 4+4, 4, 4-2 \Rightarrow 8, 4, 2, \ldots$

$t = -2 \Rightarrow x^2 = -2$ غ ق ق $\quad \alpha = \frac{1}{2} = a$

$S_7 = \frac{8(1-2^{-7})}{1-2^{-1}} = \boxed{\frac{127}{8}}$

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$S_5 = 1 + q + q^2 + q^3 + q^4 = \frac{q^5-1}{q-1} = \frac{121}{81} \Rightarrow 121q - 121 = 81q^5 - 81$

$\Rightarrow 40 = 121q - 81q^5 \Rightarrow 40 = q(121 - 3^4 q^4) \rightarrow q = \frac{1}{3}$

$\Rightarrow 40 = \frac{1}{3}(121 - 3^4 \times \frac{1}{3^4}) \Rightarrow S_5 = \frac{q^5-1}{q-1} \times 243 = \boxed{363}$

$120 \quad 40 \quad \frac{121}{81}$

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$64 = a_7 \Rightarrow 1 + 6d = 64 \Rightarrow 6d = 63 \Rightarrow d = 10,5$

$A = \text{مجموع ۴ جمله اول} = \text{جمله چهارم} \Rightarrow a_4 = 1 + 10,5 \times 3 = 32,5$

$64 = b_7 \Rightarrow b_1 r^6 = 64 \Rightarrow r^6 = 64 \Rightarrow r = 2$

$B = \text{جمله چهارم} = \text{جمله سوم} \Rightarrow b_4 = 1 \times 2^3 = 8$ $\quad A + B = 8 + 32,5 = \boxed{40,5}$

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$t_n = -24, -\frac{95}{4}, \ldots \Rightarrow d = \frac{1}{4} \Rightarrow t_n = \frac{n}{4} - \frac{97}{4}$

$-\frac{96}{4}$

$t_{101} = \frac{101}{4} - \frac{97}{4} = \frac{4}{4} = 1$

$a_n = 128, a_2, \ldots \quad a_n = 2^7 \times q^{n-1} \quad a_8 = 2^7 \times q^7 = 1$

$2^7 \quad q^7 = \frac{1}{2^7} \Rightarrow \boxed{q = \frac{1}{2}}$

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$a_1 = A \Rightarrow a_3 = A + 2d$
$a_7 = A + 6d$
$a_9 = A + 8d$

$\Rightarrow (a_7)^2 = a_3 \cdot a_9 \Rightarrow (A+6d)^2 = (A+2d)(A+8d)$
$\Rightarrow A^2 + 12dA + 36d^2 = A^2 + 10Ad + 16d^2 \Rightarrow 2Ad + 20d^2 = 0$
$\Rightarrow 2d(A+10d) = 0 \Rightarrow A + 10d = 0 \Rightarrow A = -10d$
$\hookrightarrow d = 0$ غیرقابل قبول، دنباله ثابت

$\Rightarrow \boxed{d = -\frac{a_1}{10}}$

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$a_2 = a_1 + d$
$a_4 = a_1 + 3d$
$a_8 = a_1 + 7d$
$a_2 = 2d, a_4 = 4d, a_8 = 8d$

$(a_4)^2 = a_2 \cdot a_8$ وجود جمله‌ی میانی
$\Rightarrow a_1^2 + 6a_1d + 9d^2 = a_1^2 + 8a_1d + 7d^2$
$2d^2 = 2ad \Rightarrow a = d$

$r = \frac{a_4}{a_2} = \frac{4d}{2d} = 2 \Rightarrow$ جمله‌ی عمومی $t_n = \frac{1}{4} a 2^{n-1}$

$t_{10} = \frac{1}{4_2} a 2^9 = 2^7 = \boxed{128}$

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$a_2 = aq \Rightarrow 3aq, 2aq^2, aq^3 \Rightarrow$ دنباله حسابی $\Rightarrow 2b = a + c$
$a_3 = aq^2 \Rightarrow 4aq^2 = 3aq + aq^3 \Rightarrow 4aq^2 = aq(3+q^2)$
$a_4 = aq^3 \Rightarrow q^2 + 3 - 4q = 0 \Rightarrow (q-1)(q-3) = 0$
$q = 1 \Rightarrow$ چون دنباله ثابت نیست
$\boxed{q = 3}$

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$2, \frac{7}{4}, \dots \quad a_2 = 2 + d \Rightarrow \frac{7}{4} = 2 + d \Rightarrow d = -\frac{1}{4} \quad a_n = 2 - \frac{n-1}{4}$

$a_4 = 2 - \frac{3}{4} = \frac{5}{4} \quad a_8 = 2 - \frac{7}{4} = \frac{1}{4} \quad a_{13} = 2 - \frac{12}{4} = 2 - 3 = -1$

$b_1 = \frac{5}{4} + k \quad b_2 = \frac{1}{4} + k \quad b_3 = -1 + k \Rightarrow b_2^2 = b_1 b_3$

$(\frac{1}{4} + k)^2 = (k-1)(k + \frac{5}{4}) \Rightarrow k^2 + \frac{1}{2}k + \frac{1}{16} = k^2 + \frac{1}{4}k - \frac{5}{4}$
$\Rightarrow \frac{1}{2}k - \frac{1}{4}k = -\frac{5}{4} - \frac{1}{16} \Rightarrow k = -\frac{21}{4}$

$\Rightarrow r = \frac{b_2}{b_1} = \frac{\frac{1}{4} - \frac{21}{4}}{\frac{5}{4} - \frac{21}{4}} \Rightarrow \frac{-5}{-4} = +\frac{5}{4}$
$\boxed{r = \frac{5}{4}}$

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$b_1 = a_1 \quad b_4 = a_1 r^3 \quad b_7 = a_1 r^6 \Rightarrow b_1 + b_2 + b_3 = 73 \Rightarrow a(1 + r^3 + r^6) = 73$

$A_1, A_2, A_{10} \Rightarrow A_1 = a \quad A_2 = ar^3 \quad A_{10} = ar^6 \Rightarrow A_n = A_1 + (n-1)d$

$\Rightarrow A_2 = a + d = ar^3, \quad A_{10} = a + 9d = ar^6$
$a + d = ar^3 \Rightarrow d = a(r^3 - 1)$
$a + 9d = ar^6 \Rightarrow a + 9a(r^3 - 1) = ar^6 \Rightarrow a(1 + 9(r^3 - 1)) = ar^6$
$1 + 9r^3 - 9 = r^6 \Rightarrow 9r^3 - 8 = r^6 \Rightarrow r^6 - 9r^3 + 8 = 0$
$r^3 = x \Rightarrow x^2 - 9x + 8 = 0 \Rightarrow x = \frac{9 \pm \sqrt{81 - 32}}{2} = \frac{9 \pm 7}{2}$
$x = 1 \Rightarrow r = 1$ ← چون دنباله‌ی هندسی ثابت قبول نیست
$x = 8 \Rightarrow r = 2$

$d = a(r^3 - 1) \Rightarrow d = a(7) = 7a \Rightarrow a(1 + 8 + 64) = 73 \Rightarrow 73a = 73 \Rightarrow a = 1$
$d = 7a \Rightarrow \boxed{d = 7}$