

$$a + aq + aq^2 = 21$$

$$\left\{ \begin{array}{l} a \cdot aq \cdot aq^2 = a^3 q^3 = 64 \Rightarrow aq = 4 \Rightarrow a = \frac{4}{q} \end{array} \right.$$

$$\frac{4}{q} + 4 + 4q = 21 \Rightarrow \frac{4}{q} + 4q = 17 \Rightarrow 4q^2 - 17q + 4 = 0$$

$$q = \frac{17 \pm \sqrt{289 - 64}}{8} = \frac{17 \pm 15}{8} \Rightarrow \left| q = \frac{1}{4} \right| \leq q = \frac{1}{4} \rightarrow \text{درجه اول مساوی}$$

$$n^2 + 4, 2n, n^2 - 2 \xrightarrow{\text{ضرب}} (2n)^2 = (n^2 + 4)(n^2 - 2) \Rightarrow 4n^2 = n^4 + 2n^2 - 8$$

$$2n^2 = n^4 - 8 \Rightarrow n^4 - 2n^2 - 8 = 0 \Rightarrow t = n^2 \Rightarrow t^2 - 2t - 8 = 0 \Rightarrow \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2}$$

$$\begin{array}{l} \rightarrow t = 4 \Rightarrow n^2 = 4 \Rightarrow n = 2 \Rightarrow 4+4, 4, 4-2 \Rightarrow 8, 4, 2 \dots \\ \rightarrow t = -2 \Rightarrow n^2 = -2 \end{array}$$

$$S_7 = \frac{8(1-2^{-7})}{1-2^{-1}} = \left| \frac{127}{8} \right|$$

$$S_5 = 1 + q + q^2 + q^3 + q^4 = \frac{q^5 - 1}{q - 1} = \frac{121}{81} \Rightarrow 121q - 121 = 81q^5 - 81$$

$$\Rightarrow 40 = 121q - 81q^5 \Rightarrow 40 = q(121 - 81q^4) \Rightarrow q = \frac{1}{3}$$

$$\Rightarrow 40 = \frac{1}{3} \left(\frac{121 - 81}{1 - \frac{1}{3}} \right) \Rightarrow S_5 = \frac{q^5 - 1}{q - 1} = \frac{121 - 1}{\frac{1}{3} - 1} = \frac{120}{-\frac{2}{3}} = -180$$

$$64 = a_7 \Rightarrow 1 + 6d = 64 \Rightarrow 6d = 63 \Rightarrow d = 10.5$$

$$A = \text{مجموع } a_n = \text{مجموع } b_n \Rightarrow a_4 = 1 + 10.5 \times 3 = 32.5$$

$$64 = b_7 \Rightarrow b_1 r^6 = 64 \Rightarrow r^6 = 64 \Rightarrow r = 2$$

$$B = \text{مجموع } b_n = \text{مجموع } a_n \Rightarrow b_4 = 1 \times 2^3 = 8 \quad A + B = 8 + 32.5 = 40.5$$

$$t_n = -24, -\frac{95}{4}, \dots \Rightarrow d = \frac{1}{4} \Rightarrow t_n = \frac{n}{4} - \frac{97}{4}$$

$$t_{101} = \frac{101}{4} - \frac{97}{4} = \frac{4}{4} = 1$$

$$a_n = 128, a_2, \dots \quad a_n = 2^7 \times q^{n-1} \quad a_8 = 2^7 \times q^7 = 1$$

$$q^7 = \frac{1}{2^7} \Rightarrow \left| q = \frac{1}{2} \right|$$

$a_1 = A \Rightarrow a_3 = A + 2d \Rightarrow (a_7)^2 = a_3 \cdot a_9 \Rightarrow (A + 6d)^2 = (A + 2d)(A + 8d)$
 $A + 6d = A + 6d \Rightarrow A^2 + 12Ad + 36d^2 = A^2 + 10Ad + 16d^2 \Rightarrow 2Ad + 20d^2 = 0$
 $A + 8d = A + 8d \Rightarrow 2d(A + 10d) = 0 \Rightarrow A + 10d = 0 \Rightarrow A = -10d$
 $\hookrightarrow d = 0$ ~~مقدار د برابر صفر است~~
 $\Rightarrow d = -\frac{a_1}{10}$ سوال تلفظی مقدار د برابر صفر است ✓

$a_2 = a_1 + d$ $(a_4)^2 = a_2 \cdot a_8 \Rightarrow a_1^2 + 6ad + 9d^2 = a_1^2 + 8ad + 7d^2$
 $a_4 = a_1 + 3d$ مقدار د برابر صفر است $2d^2 = 2ad \Rightarrow d = d$
 $a_8 = a_1 + 7d$
 $\downarrow$ $a_2 = 2d, a_4 = 4d, a_8 = 8d$ $r = \frac{a_4}{a_2} = \frac{4d}{2d} = 2 \Rightarrow t_n = \frac{1}{4} \cdot 2^{n-1}$
 $t_{10} = \frac{1}{4} \cdot 2^9 = 2^7 = 128$ ✓

$a_2 = aq \Rightarrow 3aq, 20aq^2, aq^3 \Rightarrow 3b = a + c$
 $a_3 = aq^2 \Rightarrow 40aq^3 = 3aq + aq^3 \Rightarrow 40d^2 = 0d(3 + q^2)$
 $a_4 = aq^3 \Rightarrow q^2 + 3 - 4q = 0 \Rightarrow (q-1)(q-3) = 0 \Rightarrow q = 1$ ✓
 $q = 3$ ✓

$2, \frac{7}{4}, \dots$ $a_2 = 2 + d \Rightarrow \frac{7}{4} = 2 + d \Rightarrow d = -\frac{1}{4}$ $a_n = 2 - \frac{n-1}{4}$
 $a_4 = 2 - \frac{3}{4} = \frac{5}{4}$ ✓ $a_8 = 2 - \frac{7}{4} = \frac{1}{4}$ $a_{13} = 2 - \frac{12}{4} = 2 - 3 = -1$
 $b_1 = \frac{5}{4} + k$ $b_2 = \frac{1}{4} + k$ $b_3 = -1 + k \Rightarrow b_2^2 = b_1 b_3$
 $(\frac{1}{4} + k)^2 = (k-1)(k + \frac{5}{4}) \Rightarrow k^2 + \frac{1}{2}k + \frac{1}{16} = k^2 + \frac{1}{4}k - \frac{5}{4}$
 $\Rightarrow r = \frac{b_2}{b_1} = \frac{\frac{1}{4} - \frac{21}{4}}{\frac{5}{4} - \frac{21}{4}} = \frac{-5}{-16} = \frac{5}{16}$ ✓

$b_1 = a_1$ $b_4 = ar^3$ $b_7 = ar^6 \Rightarrow b_1 + b_2 + b_3 = 73 \Rightarrow a(1 + r^3 + r^6) = 73$
 $A_1, A_2, A_{10} \Rightarrow A_1 = a$ $A_2 = ar^3$ $A_{10} = ar^9 \Rightarrow A_n = A_1 + (n-1)d$
 $\Rightarrow A_2 = a + d = ar^3$ $A_{10} = a + 9d = ar^9$ $a + d = ar^3 \Rightarrow d = a(r^3 - 1)$ ✓
 $a + 9d = ar^9 \Rightarrow a + 9a(r^3 - 1) = ar^9 \Rightarrow a(1 + 9(r^3 - 1)) = ar^9$
 $1 + 9r^3 - 9 = r^9 \Rightarrow 9r^3 - 8 = r^9 \Rightarrow r^9 - 9r^3 + 8 = 0$
 $r^3 = x \Rightarrow x^3 - 9x + 8 = 0 \Rightarrow x = 1$ ✓
 $x = 8 \Rightarrow r^3 = 8 \Rightarrow r = 2$
 $d = a(r^3 - 1) \Rightarrow d = a(8 - 1) = 7a \Rightarrow a(1 + 8 + 64) = 73 \Rightarrow 73a = 73 \Rightarrow a = 1$
 $d = 7a \Rightarrow d = 7$ ✓