

$$S_n = a_1 \left(\frac{q^n - 1}{q - 1} \right) \Rightarrow S_3 = a_1 \left(\frac{q^3 - 1}{q - 1} \right) = 21$$

$$a_1 q + a_1 q^2 = a_1 q^3 \Rightarrow a_1 q^2 = \frac{q^3}{q^2} \Rightarrow a_1 = \frac{q}{q^2}$$

$$\Rightarrow \frac{q}{q^2} \left(\frac{q^3 - 1}{q - 1} \right) = 21 \Rightarrow \frac{q^3 - q}{q - 1} = 21 \Rightarrow q^3 - q = 21q - 21 \Rightarrow q = \cancel{X} \cdot \cancel{X} \cdot \boxed{\frac{1}{q}}$$

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$$b^2 = ac \Rightarrow (2x)^2 = (x^2 - 2)(x^2 + 4) \Rightarrow 4x^2 = x^4 - 1 + 2x^2 \Rightarrow 2x^2 = x^4 - 1 \Rightarrow x^4 - 2x^2 - 1 = 0$$

$$\Rightarrow x = 2 \Rightarrow a_1 = x^2 + 2 = 4 + 2 = 6 \Rightarrow q = \frac{1}{4}$$

$$S_5 = 1 \left(\frac{\left(\frac{1}{4}\right)^5 - 1}{\frac{1}{4} - 1} \right) = 1 \left(\frac{\frac{1}{1024} - 1}{-\frac{3}{4}} \right) = 1 \left(\frac{\frac{1 - 1024}{1024}}{-\frac{3}{4}} \right) = 1 \times \frac{127}{94} = \boxed{\frac{127}{94}}$$

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$$a + aq + aq^2 + aq^3 + aq^4 = a(1 + q + q^2 + q^3 + q^4) = 233 \times \frac{121}{11} = \boxed{363}$$

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$$d = \frac{94 - 1}{a + 1} = \frac{93}{a + 1} = 101 \Rightarrow t_f = a + 3d = 1 + 311 = 312 = A$$

$$q^{n+1} = \frac{b}{a} \Rightarrow q^5 = \frac{94}{1} \Rightarrow q^5 = 94 \Rightarrow q = 2 \Rightarrow a_5 = aq^4 = 1 \times 2^4 = 16 = B$$

$$\Rightarrow A + B = 312 + 16 = \boxed{328}$$

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$$d = \frac{-9\omega + 24}{2 - 1} = -9\omega + 24 = \frac{-9\omega + 96}{4} = \frac{1}{4} = 0.25 \Rightarrow t_{b1} = a_1 + 100d = -24 + 25 = 1$$

$$q^{1-1} = \frac{1}{128} \Rightarrow q^0 = \frac{1}{128} \Rightarrow q^0 = \frac{1}{2^7} \Rightarrow q = \boxed{\frac{1}{2}}$$

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$$\begin{cases} a_r = b_1 \\ a_v = b_r \\ a_q = b_r \end{cases}$$

b_1, b_r, b_q و b_d دینا لای حتمی مستند.

$$b_r = b_1, b_r \Rightarrow (a + qd)^r = (a + rd)(a + ld) \Rightarrow a^r + r q d^r + l r d a = a^r + l r d^r + l o d a$$

$$\Rightarrow r o d + l r d a = 0 \Rightarrow l o d + d a = 0 \Rightarrow d(l o d + a) = 0 \Rightarrow \begin{cases} d = 0 \\ l o d + a = 0 \Rightarrow a = -l o d \Rightarrow d = \begin{bmatrix} a_1 \\ -l o \end{bmatrix} \end{cases}$$

$$\begin{cases} a_r = b_1 \\ a_v = b_r \\ a_n = b_r \end{cases}$$

b_1, b_r, b_q و b_d دینا لای حتمی مستند.

$$b_r = b_1, b_r \Rightarrow (a + rd)^r = (a + d)(a + vl) \Rightarrow a^r + r d^r + v d a = a^r + v d^r + l d a \Rightarrow r d^r - v d a = 0$$

$$\Rightarrow d^r - d a = 0 \Rightarrow d(d - a) = 0 \xrightarrow{d \neq 0 \Rightarrow d = a} d - a = 0 \Rightarrow d = a$$

$$\frac{b_r}{b_1} = \frac{a + rd}{a + d} = \frac{d + rd}{d + d} = \frac{rd}{2d} = r = q \rightarrow a_1 = \frac{1}{r} \Rightarrow a_b = a q^9 = \frac{1}{r} \times r^9 = r^8 = \boxed{128}$$

$$\begin{matrix} r a q & r a q^r & a q^r \\ \downarrow & \downarrow & \downarrow \\ b_1 & b_r & b_r \end{matrix}$$

b_1, b_r, b_q و b_d دینا لای حتمی مستند.

$$r b_r = b_1 + b_r \Rightarrow r(r a q^r) = (r a q) + (a q^r) \Rightarrow r a q^r = a q(r + q^r) \Rightarrow r q = r + q^r$$

$$\Rightarrow q^r + r - r q = 0 \Rightarrow q = \boxed{r}$$

$$d = \frac{v - r}{r - 1} = \frac{v}{r} - r = -\frac{1}{r} = -0.125$$

$$t_f = a_1 + rd = r - 0.125 = 1.125$$

$$t_n = a_1 + vd = r - 1.125 = 0.125$$

$$t_{1r} = a_1 + ld = r - r = -1$$

$$1.125 + x, 0.125 + x, -1 + x \rightarrow b^r = ac \Rightarrow \left(\frac{1}{r} + x\right)^r = \left(\frac{a}{r} + x\right)(-1 + x) \Rightarrow \frac{1}{19} + x^r + \frac{x}{r} = -\frac{a}{r} + x^r + \frac{1}{r}x \Rightarrow \frac{1}{19} + \frac{x}{r} = -\frac{a}{r} + \frac{1}{r}x \Rightarrow \frac{x}{r} = -\frac{r_0}{19} - \frac{1}{19} = -\frac{r_1}{19} \Rightarrow x = \frac{-r_1}{r} = 0.125$$

$$\begin{matrix} a + a q^r + a q^9 = v r \\ \downarrow \quad \downarrow \quad \downarrow \\ t_1 \quad t_r \quad t_o \end{matrix}$$

$$\Rightarrow a + a + d + a + qd = vr \Rightarrow 3a + lod = vr \Rightarrow a = 1, d = \boxed{v}$$