

$$\left. \begin{aligned} \frac{a}{q} + a + aq &= 21 \Rightarrow a \left(\frac{q^2 + q + 1}{q} \right) = 21 \\ \frac{a}{q} \times a \times aq &= 27 \Rightarrow a^3 = 27 \Rightarrow a = 3 \end{aligned} \right\} \Rightarrow \frac{q^2 + q + 1}{q} = \frac{21}{3} \Rightarrow$$

$$q^2 + q + 1 = 7q \Rightarrow q^2 - 6q + 1 = 0 \Rightarrow (q-1)(q-16) = 0 \Rightarrow \begin{cases} q = \frac{1}{16} \\ q = 16 \end{cases} \Rightarrow$$

$$b^2 = ac \Rightarrow (2a)^2 = (a^2 + 4)(a^2 - 2) \Rightarrow 4a^2 = a^4 + 2a^2 - 2 \Rightarrow$$

$$a^4 - 2a^2 - 2 = 0 \Rightarrow (a^2 - 4)(a^2 + 2) = 0 \Rightarrow \begin{cases} a^2 = -2 \\ a^2 = 4 \end{cases} \Rightarrow a = \pm 2$$

$$\begin{aligned} \text{اگر } a = 2: & \quad 1, 4, 16 \\ \text{اگر } a = -2: & \quad 1, -4, 16 \end{aligned} \Rightarrow a = 2, q = \frac{1}{16}, a_1 = 1$$

$$S_n = a_1 \times \frac{1 - q^n}{1 - q} = 1 \times \frac{1 - (\frac{1}{16})^4}{1 - \frac{1}{16}} = \frac{125}{16}$$

$$q^0 + q^1 + q^2 + q^3 + q^4 = \frac{121}{16} \Rightarrow q^0 \times \frac{(q^5 - 1)}{q - 1} = \frac{121}{16} \Rightarrow$$

$$\frac{q^5 - 1}{q - 1} = \frac{121}{16}$$

$$S_5 = a_1 \times \frac{(q^5 - 1)}{q - 1} = \frac{121}{16} \times \frac{16}{15} = \frac{121}{15}$$

$$\begin{aligned} \text{رابطه حسابی} & \Rightarrow \begin{cases} a_1 = 1 \\ a_5 = 25 \end{cases} \Rightarrow 25 = a_1 + 4d \Rightarrow d = 6 \Rightarrow A = \frac{4d}{2} = 12 \\ \text{رابطه هندسی} & \Rightarrow \begin{cases} a_1 = 1 \\ a_5 = 25 \end{cases} \Rightarrow 25 = a_1 \times q^4 \Rightarrow q = \pm 1 \Rightarrow B = \pm 1 \end{aligned}$$

$$A + B = 12 + 1 = 13$$

$$A + B = 12 - 1 = 11$$

$$-24, -\frac{9}{2}, \dots, a_{101} = -24 + (100 \times \frac{1}{2}) = 1$$

$$128, a_2, \dots, a_n = 2 \times \frac{1}{2} = 1 \Rightarrow (2q)^n = 1 \Rightarrow q = \frac{1}{2}$$

$$a_3 = a_1 + 2d$$

$$a_4 = a_1 + 3d$$

$$a_5 = a_1 + 4d$$

(2) ✓

$$\frac{a_1 + 2d}{a_1 + 3d} = \frac{a_1 + 4d}{a_1 + 5d} \Rightarrow$$

$$a_1^2 + 4d^2 + 12a_1d = a_1^2 + 10a_1d + 14d^2 \Rightarrow$$

$$4d^2 = -20a_1d \Rightarrow$$

دنباله ثابت می شود $\rightarrow$ اگر $d = 0$ اگر $d \neq 0 \rightarrow 2a_1 = -20d \Rightarrow d = -\frac{a_1}{10}$

اگر $d \neq 0$ باشد مقدار آن برابر $-\frac{a_1}{10}$ می باشد

$$a_2 = a_1 + d$$

$$a_3 = a_1 + 2d$$

$$a_4 = a_1 + 3d$$

$$b^2 = ac \Rightarrow (a_1 + 2d)^2 = (a_1 + d)(a_1 + 3d) \Rightarrow$$

$$a_1^2 + 4d^2 + 4a_1d = a_1^2 + 4a_1d + 3d^2 \Rightarrow$$

$$d^2 = 0 \Rightarrow d = 0 \Rightarrow a_n = a_1 + (n-1)d \Rightarrow$$

زیرا اگر $d = 0$ باشد، سه جمله دنباله برابر می شود و شرط چهارم برآورده نمی شود. سوال نقض می شود.

(2) ✓

$$\frac{a_n}{a_1} = \frac{10a_1}{a_1} = 10 \Rightarrow q = 10$$

$$\frac{a_2}{a_1} = \frac{2a_1}{a_1} = 2 \Rightarrow q = 2$$

$$a_n = a_1 q^{n-1} = \frac{1}{2} \times 2^9 = 2^8 = 128$$

$$2b = a + c \Rightarrow 2 \times 2a_p = 2a_1 + a_q \Rightarrow 2a_p q = a_1(2 + q^2) \Rightarrow$$

$$(2q) = a_1(2 + q^2) \xrightarrow{a_1 \neq 0} 2q = 2 + q^2 \Rightarrow q^2 - 2q + 2 = 0 \Rightarrow$$

چون قدرنسب دنباله حسابی و جمله اولش می تواند صفر باشد. اگر $a_1 = 0$ باشد دنباله هندسی می تواند شکل می شود.

$$(q-1)(q-2) = 0 \Rightarrow$$

$$q = 1 \text{ یا } q = 2 \Rightarrow q = 2$$

$$a_1 = 2$$

$$a_2 = 2 + 2(-\frac{1}{2}) = \frac{1}{2}$$

$$d = -\frac{1}{2}$$

$$a_3 = 2 + 2(-\frac{1}{2}) = 1$$

$$a_4 = 2 + 2(-\frac{1}{2}) = -1$$

$$b^2 = ac \Rightarrow (\frac{1}{2} + 2)^2 = (\frac{1}{2} + 2)(-1 + 2)$$

$$\frac{1}{4} + 2 + 1 = 2 + \frac{1}{2} - 1 \Rightarrow$$

$$\frac{1}{4} = -\frac{1}{4} \Rightarrow a = -\frac{21}{4}$$

$$q = \frac{-1 + (-\frac{21}{4})}{\frac{1}{2} + (-\frac{21}{4})} = \frac{-\frac{22}{4}}{-\frac{20}{4}} = \frac{11}{10}$$

(2) ✓

$$a_4^2 = a_1 \times a_9 \Rightarrow (a_1 + 3d)^2 = a_1(a_1 + 8d) \Rightarrow$$

$$a_1^2 + 6d^2 + 6a_1d = a_1^2 + 8a_1d \Rightarrow d = \frac{2}{3}a_1$$

$d \neq 0$

$$d = \frac{2}{3}a_1$$

$$a_1 + a_1 + d + a_1 + 9d = 10 \Rightarrow a_1 + a_1 + \frac{2}{3}a_1 + 9 \times \frac{2}{3}a_1 = 10 \Rightarrow$$

(1, 10)

$$10a_1 = 10 \Rightarrow a_1 = 1 \Rightarrow d = \frac{2}{3}$$