نام و نام خانوادگی آرتین شیروانی فر ...... پاسخنامه تشریحی تکلیف شماره ۱۵ ... کلاس دهم ریاضی B ......

۱

$a \times aq \times aq^2 = 64 \rightarrow a^3q^3 = 64 \rightarrow aq = 4$

$a(1+q+q^2) = 21$

$\rightarrow \dfrac{1+q+q^2}{aq^2} = \dfrac{21}{16} \rightarrow \dfrac{1+q+q^2}{4q} = \dfrac{21}{16}$

$(aq \cdot q) = 4q$

$16q^2 - 68q + 16 = 0$

$4q^2 - 17q + 4 = 0$

$\rightarrow q^2 - 17q + 16 = 0$

$(q-1)(q-16) = 0 \rightarrow q=1 ,\ q=16$

$\div 4$

غیر قابل قبول $q = 4$, غیر صعودی $q = \frac{1}{4}$ جواب

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در دنباله هندسی

$(n^2+4)(n^2-2) = 4n^2 \rightarrow n^4 + 2n^2 - 8 = 4n^2 \rightarrow n^4 - 2n^2 - 8 = 0$

$(n^2-4)(n^2+2) = 0 \begin{cases} n^2 = 4 \\ n^2 = -2 \text{ غ ق ق} \end{cases}$

$a_1 = n^2+4$

$a_2 = 2n$

$a_3 = n^2-2$

$n^2 = 4 \rightarrow n = \pm 2$

$n = 2 \rightarrow a_1 = 8,\ a_2 = 4,\ a_3 = 2$ ✓ $\rightarrow q = \frac{1}{2}$

$n = -2 \rightarrow a_1 = 8,\ a_2 = -4,\ a_3 = 2$ X

$S_7 = a_1 \dfrac{\left(1 - \left(\frac{1}{2}\right)^7\right)}{1-\frac{1}{2}} = 8 \times \dfrac{\frac{127}{128}}{\frac{1}{2}} = 8\left(\frac{127}{64}\right) = \frac{127}{8} = 15.875$

۳

دو طرف را در a_1 ضرب می‌کنیم:

$S_5 = a_1(1+q+q^2+q^3+q^4) = \underbrace{\frac{121}{81} \times a_1}$

$\frac{121}{\cancel{81}} \times \cancel{243}^{3} = 363$

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$A = \dfrac{64+1}{2} = 32.5$

$B^2 = 64 \times 1 \rightarrow B = \pm\sqrt{64 \times 1} \rightarrow B = \pm 8$

$\begin{cases} A = 32.5 \\ B = 8 \end{cases} \rightarrow A + B = 40.5$ (۱)

(۲) جواب

$\begin{cases} A = 32.5 \\ B = -8 \end{cases} \rightarrow A + B = 24.5$

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$a_n = -24 ,\ -23.75 ,\ \ldots \rightarrow a_n = \frac{1}{4}n - 24.25$

$d = +0.25$

$a_{101} = \underbrace{\frac{1}{4} \times 101}_{25.25} - 24.25 = 1$

$b_n = 128 ,\ a_2 ,\ \ldots ,\ \cancel{a_8}^{1} \rightarrow q^7 = \frac{1}{128}$

$q = \frac{1}{2}$

$$\left.\begin{array}{l} a_3 = a_1 + 2d \\ a_7 = a_1 + 6d \\ a_9 = a_1 + 8d \end{array}\right\} \rightarrow (a_1+6d)^2 = (a_1+2d)(a_1+8d)$$ ۶

$$\cancel{a^2} + 12ad + 36d^2 = \cancel{a^2} + 10ad + 16d^2$$

$$2ad = -20d^2 \rightarrow a = -10d \rightarrow d = -\frac{a}{10}$$

$$2ad + 20d^2 = 0 \rightarrow 2d(a+10d) = 0 \rightarrow d = 0$$

هم می‌تواند باشد چون دنباله‌ی ثابت هم ممکن است.

$$\left.\begin{array}{l} a_2 = a+d \\ a_4 = a+3d \\ a_8 = a+7d \end{array}\right\} \rightarrow (a+d)(a+7d) = (a+3d)^2$$ ۷

$$\cancel{a^2} + 8ad + 7d^2 = \cancel{a^2} + 6ad + 9d^2$$

$$2ad = 2d^2 \rightarrow ad = d^2 \rightarrow a = d$$

$$2ad - 2d^2 = 0 \rightarrow 2d(a-d) = 0$$

$$2d = 0 \rightarrow d = 0$$

غیر قابل قبول

چون دنباله ثابت می‌شود.

$$2d \xrightarrow{\times 2} 4d,\ 8d$$

$$a = 2 \rightarrow a_{10} = \frac{1}{4} \times 2^9 = 128$$

$$3aq,\ 2aq^2,\ aq^3 \rightarrow$$ دنباله‌ی حسابی غیر ثابت ۸

$$2aq^2 = \frac{3aq + aq^3}{2} \rightarrow 4aq^2 = 3aq + aq^3$$

$$\cancel{aq}(4q) = \cancel{aq}(3+q^2)$$

$$4q = 3 + q^2 \rightarrow q^2 - 4q + 3 = 0$$

$$(q-3)(q-1) = 0 \rightarrow q = 1 \times,\quad \boxed{q = 3} \checkmark$$

چون گفته غیر ثابت $q=1$ نمی‌تواند باشد

$$(x + a_4)(x + a_{13}) = (x + a_8)^2$$

$$a_n = 2, \frac{7}{4}, \ldots \qquad d = -\frac{1}{4} \qquad a_n = -\frac{1}{4}n + \frac{9}{4}$$

$$a_4 = -\frac{1}{4}\times 4 + \frac{9}{4} = \frac{5}{4}$$

$$a_8 = -\frac{1}{4}\times 8 + \frac{9}{4} = \frac{1}{4}$$

$$a_{13} = -\frac{1}{4}\times 13 + \frac{9}{4} = -1$$

$$\left(x + \frac{5}{4}\right)(x - 1) = \left(x + \frac{1}{4}\right)^2$$

$$\cancel{x^2} + \frac{1}{4}x - \frac{5\times 4}{4\times 4} = \cancel{x^2} + \frac{1\times 2}{2\times 2}x + \frac{1}{16}$$

$$\frac{1}{4}x = -\frac{21}{16} \rightarrow x = -\frac{21}{4}$$

$$a_n = -4 \xrightarrow{\times\frac{5}{4}} -5 \xrightarrow{\times\frac{5}{4}} -\frac{25}{4} \rightarrow q = \frac{5}{4}$$

$$a + aq^3 + aq^6 = 73 \rightarrow \left.\begin{array}{l} a = a \\ a + d = aq^3 \\ a + 9d = aq^6 \end{array}\right\} \rightarrow (a+d)^2 = a(a+9d)$$ ۱۰

$$\cancel{a^2} + 2ad + d^2 = \cancel{a^2} + 9ad$$

$$d^2 - 7ad = 0$$

$$d(d - 7a) = 0 \rightarrow d - 7a = 0,\quad d = 0$$

دنباله‌ی ثابت ✓

$$d - 7a = 0 \rightarrow a = \frac{1}{7}d$$

$$\frac{1}{7}d + \frac{8}{7}d + \frac{64}{7}d = 73$$

$$\frac{73}{7}d = 73 \rightarrow d = 7 \checkmark$$

$$d = 7$$

$$d = 0$$