

$$a_1 + a_2 + a_3 = 21$$

$$a_1 + a_1q + a_1q^2 = 21$$

$$a_1 \times a_1q \times a_1q^2 = 27 \Rightarrow a_1^3 \times q^3 = 27 \Rightarrow a_1q = 3 \Rightarrow a_1 = \frac{3}{q} \Rightarrow$$

$$a_1 + a_1q + a_1q^2 = 21 \Rightarrow \frac{3}{q} + 3 + 3q = 21 \Rightarrow 3 + 3q + 3q^2 = 21q \Rightarrow 3q^2 - 18q + 3 = 0 \Rightarrow q^2 - 6q + 1 = 0 \Rightarrow (q-1)(q-5) = 0 \Rightarrow q = 1 \text{ یا } q = 5$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} \Rightarrow a_1 \times a_3 = (a_2)^2 \Rightarrow 3 \times 9 = (3q)^2 \Rightarrow 27 = 9q^2 \Rightarrow q^2 = 3 \Rightarrow q = \sqrt{3} \text{ یا } q = -\sqrt{3}$$

$$q^2 - 3 = 0 \Rightarrow (q - \sqrt{3})(q + \sqrt{3}) = 0 \Rightarrow q = \sqrt{3} \text{ یا } q = -\sqrt{3}$$

$$q = \pm \sqrt{3} \Rightarrow \text{if } q = \sqrt{3} \Rightarrow \text{جواب: } 1, 3, 9, \dots \Rightarrow q = \frac{1}{\sqrt{3}} \rightarrow q > 0 \Rightarrow \text{قبول}$$

$$\text{if } q = -\sqrt{3} \Rightarrow \text{جواب: } 1, -3, 9, \dots \Rightarrow q = -\frac{1}{\sqrt{3}} \rightarrow q < 0 \Rightarrow \text{غیر قابل قبول}$$

$$S_4 = a_1 \left(\frac{1 - q^4}{1 - q} \right) = 1 \times \left(\frac{1 - \left(\frac{1}{\sqrt{3}} \right)^4}{1 - \frac{1}{\sqrt{3}}} \right) = 1 \times \left(\frac{1 - \frac{1}{9}}{1 - \frac{1}{\sqrt{3}}} \right) = 1 \times 2 \times \frac{12\sqrt{3}}{12\sqrt{3} - 12} = \frac{12\sqrt{3}}{12\sqrt{3} - 12} = \frac{12\sqrt{3}}{12(\sqrt{3} - 1)} = \frac{\sqrt{3}}{\sqrt{3} - 1}$$

$$S_5 = a_1 + a_2 + a_3 + a_4 + a_5 = a_1 + a_1q + a_1q^2 + a_1q^3 + a_1q^4 \Rightarrow S_5 =$$

$$a_1(1 + q + q^2 + q^3 + q^4) \Rightarrow a_1 = 27 \text{ و } 1 + q + q^2 + q^3 + q^4 = \frac{121}{27} \Rightarrow$$

$$S_5 = 27 \times \frac{121}{27} = 121 \Rightarrow \boxed{121}$$

$$S_5 = a \left(\frac{1 - q^5}{1 - q} \right) \Rightarrow 1 - q^5 \div 1 - q = (q^4 + q^3 + q^2 + q + 1) \Rightarrow \frac{1 - q^5}{1 - q} = \frac{121}{27} \Rightarrow S_5 = 27 \times \frac{121}{27} = \boxed{121}$$

$$1, -3, 9, -27, 81, -243, \dots / 1, -3, 9, -27, 81, -243, \dots$$

$$A = a_1 \times \frac{1 - q^4}{1 - q} \Rightarrow a_1 + qd = 2a_1 \Rightarrow 4r + 1 = 2a_1 \Rightarrow a_1 = \frac{4r + 1}{2}$$

$$B = a_1 \times \frac{1 - q^4}{1 - q} = a_1 \times a_1 \times q^4 = 4r \Rightarrow a_1 = \pm 1 \begin{cases} B + A = \frac{4r}{1} + 1 = \frac{11}{1} = 12 \\ A + B = \frac{4r}{1} - 1 = \frac{9}{1} = 10 \end{cases}$$

$$d = -\frac{4r}{1} - (-4r) = -\frac{4r}{1} + \frac{4r}{1} = 0 \Rightarrow a_{101} = a_1 + 100d = a_1 + \frac{100}{1}$$

$$a_2 = -\frac{4r}{1}$$

$$\Rightarrow a_{101} = \frac{-4r}{1} + \frac{100}{1} = \frac{r}{1} = 1$$

$$a_1 = -4r$$

$$a_1 = 121$$

$$a_n = 121 \times q^n \Rightarrow a_n = 1 = 121 \times q^n \Rightarrow \boxed{q = \frac{1}{121}}$$

$$a_r, a_{r+1}, a_{r+2} = a+rd, a+(r+1)d, a+(r+2)d \Rightarrow (a+rd)^r = (a+rd)(a+(r+1)d) \Rightarrow$$

$$a^r + r^2 ad + r^2 d^2 = a^r + r^2 ad + r^2 d^2 \Rightarrow r^2 ad + r^2 d^2 = 0 \Rightarrow r^2 d(a+d) = 0$$

$$\Rightarrow \boxed{d=0} \vee d = \frac{-a}{10} = \boxed{-\frac{1}{10}a}$$

$$\frac{a+rd}{a+d} = \frac{a+rd}{a+rd} \Rightarrow (a+rd)^r = (a+d)(a+rd) \Rightarrow$$

$$a^r + r^2 ad + r^2 d^2 = a^r + r^2 ad + r^2 d^2 \Rightarrow -r^2 ad + r^2 d^2 = 0 \Rightarrow a=d$$

$$a_r = b_1 = \frac{1}{r} \Rightarrow a+d = \frac{1}{r} \Rightarrow a = \frac{1}{r} \Rightarrow a = \frac{1}{r} = d \Rightarrow$$

$$q = \frac{a_r}{a_1} = \frac{a+rd}{a+d} = \frac{\frac{1}{r}}{\frac{1}{r}} = 1 \Rightarrow a_{10} = \frac{1}{r} \times q = \frac{1}{r} = \boxed{\frac{1}{10}}$$

$$r a_r, r a_{r+1}, r a_{r+2} \rightarrow r(r a_r) = r^2 a_r + a_r \Rightarrow r^2 a_r + a_r = r^2 a_{r+1} + a_{r+1} \Rightarrow$$

$$r^2 a_r = r^2 a_{r+1} + a_r \Rightarrow r^2 a_r - r^2 a_{r+1} - a_r = 0 \Rightarrow r^2 (a_r - a_{r+1}) - a_r = 0 \Rightarrow r^2 (a_r - a_{r+1}) = a_r$$

$$\Rightarrow q = 0 \vee 3 \vee 1 \rightarrow q = 3 \rightarrow \text{فقط قابل قبول است}$$

$$\frac{1}{r}, \frac{1}{r+1}, \frac{1}{r+2}, \dots \Rightarrow a_r = r + r \times \left(-\frac{1}{r}\right) = \frac{1}{r} \text{ و } a_{r+1} = r + (r+1) \times \left(-\frac{1}{r}\right) = \frac{1}{r+1}$$

$$a_{r+2} = r + (r+2) \times \left(-\frac{1}{r}\right) = \frac{1}{r+2}$$

$$b_1 = a_{r+2} + k, b_r = a_{r+1} + k, b_{r+1} = a_r + k \Rightarrow (b_r)^r = b_r \times b_1 \Rightarrow$$

$$\left(\frac{1}{r} + k\right)^r = \left(\frac{1}{r+1} + k\right) \left(-\frac{1}{r+2} + k\right) \Rightarrow \frac{1}{r^r} + \frac{1}{r} k + k^r = \frac{1}{r+1} k - \frac{1}{r+2} + k^2 - k \Rightarrow$$

$$k \left(\frac{1}{r^r} + \frac{1}{r} k - \frac{1}{r+1} k + \frac{1}{r+2}\right) = 0 \Rightarrow 1 + 1k - rk = 0 \Rightarrow k = \frac{-1}{r} \Rightarrow q = \frac{b_r}{b_1} = \frac{\frac{1}{r+1} + \frac{1}{r+2}}{\frac{1}{r} + \frac{1}{r+1}} = \frac{1}{r} \Rightarrow$$

$$a_1, a_2, a_{10} = q, aq^r, aq^y \Rightarrow$$

$$a_r - a_1 = \frac{a_{10} - a_1}{10} \Rightarrow aq^r - a = \frac{aq^y - aq^r}{10} = \frac{aq^r(aq^{y-r} - 1)}{10} \Rightarrow$$

$$q^y - qq^r + 1 = 0 \Rightarrow (q^y - 1)(q^r - 1) = 0 \Rightarrow q = 1 \vee (q = 2) \Rightarrow$$

$$a + a(r)^r + a(r)^y = vr \Rightarrow a(1 + r^r + r^y) = vr \Rightarrow a = \frac{vr}{1 + r^r + r^y} = 1 \Rightarrow d = a_r - a_1 = 1 - 1 = 0$$