۷-

$q = 2$, $a_1 = \frac{1}{4}$

$a_n = \frac{1}{4} \times 2^{n-1}$

$a_{10} = \frac{1}{4} \times 2^{9} = \boxed{128}$

$a_2 , \quad a_2 + 2d , \quad a_2 + 4d$

$(a_2+2d)^2 = (a)(a+4d) \to a^2 + 4d^2 + 4ad = a^2 + 4ad$

$4d^2 = 2ad$

$2d = a$

۸-

$3a_2 , 2a_3 , a_4$

$3(aq), 2(aq^2), aq^3 \to 2aq^2 = \frac{3aq + aq^3}{2} \to -4aq^2 + 3aq + aq^3 = 0 \to aq(-4q + 3 + q^2)$

$aq = 0$, $a = 0$ غیر قابل قبول است

$(q-1)(q-3) = 0$

$q = 1$ غ ق ق

$\boxed{q = 3}$

۹-

$d = \frac{7}{4} - 2 = -\frac{1}{4} \to a_4 = 2 - \frac{3}{4} = \frac{5}{4}$, $a_8 = 2 - \frac{7}{4} = \frac{1}{4}$, $a_{13} = 2 - \frac{12}{4} = -1$

$\frac{5}{4} + x , \frac{1}{4} + x , -1 + x$

$(\frac{1}{4} + x)^2 = (x + \frac{5}{4})(x - 1)$

$\frac{1}{16} + x^2 + \frac{1}{2}x = x^2 + \frac{1}{4}x - \frac{5}{4} \to x = -$

$-4 , -5 , -\frac{25}{4} \to q = \frac{-\frac{25}{4}}{-\frac{5}{1}} = \boxed{\frac{5}{4}}$

۱۰-

$a + aq^3 + aq^4 = 73$

$t_1 = a_1 = a , t_2 = a_4 = aq^3 , t_{10} = a_7 = aq^4$

$t_2 = aq^3$, $t_2 = t_1 + d$ | $aq^3 = a + d \to d = a(q^3 - 1)$

$t_{10} = aq^4 = a + 9a(q^3 - 1) = a + 9aq^3 - 9a \to aq^4 - 9aq^3 + 8a = 0 \to$

$a(q^4 - 9q^3 + 8) = 0 , q^3 = \frac{9 \pm \sqrt{81 - 32}}{2}$ ① $q^3 = 8 \to q = 2$ ② $q^3 = 1 \to q = 1$

$q = 1 \to a + a + a = 73$

$\to a = \frac{73}{3} \to d = a(q^3 - 1) = \frac{73}{3} \times (1 - 1) = \boxed{0}$

$q = 2$

$\to a + 8a + 64a = 73 \to a = 1 \to d = a(q^3 - 1) = 1 \times (8 - 1) = 7$

۱- امیدی/روش

$a_1 + a_2 + a_3 = 21$

$a_1 \times a_2 \times a_3 = 64 \rightarrow \frac{a}{q} + a + aq = 64 \rightarrow a^3 = 64$, $a = 4$

$\frac{4}{q} + 4 + 4q = 21$

$\frac{4}{q} + 4q = 17 \rightarrow \frac{4q^2+4}{q} = 17 \rightarrow 17q = 4 + 4q^2$

$4q^2 - 17q + 4 = 0 \rightarrow \Delta = 289 - 64 = 225 \rightarrow q$ — عمومی

$\rightarrow \frac{17-15}{8} = \boxed{\frac{1}{4}}$

$\rightarrow \frac{17+15}{8} = 4$

۲-

$(2x)^2 = (x^2-2)(x^2+4) \longrightarrow 8, 4, 2$ $\quad \times\frac{1}{2} \quad \times\frac{1}{2}$ $\qquad q = \frac{1}{2}$

$4x^2 = x^4 + 2x^2 - 8$

$2x^2 = x^4 - 8 \rightarrow x^4 - 2x^2 - 8 = 0 \rightarrow \underbrace{(x^2-4)}(x^2+2) = 0$

$x^2 = 4$

$x = \pm 2$

$S_7 = \left(\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}\right) 8 = \left(1 - \frac{1}{128}\right) \times 16 = \frac{127}{\cancel{128}_{8}} \times \cancel{16} = \frac{127}{8} = \boxed{15\frac{7}{8}}$

۳-

$1 + q + q^2 + q^3 + q^4 = \frac{121}{81} \longrightarrow$ بدیهی است که اگر در جمله اول ضرب شود حاصل ۵ جمله اول را به ما می دهد

$243(1+q+q^2+q^3+q^4) = \frac{121}{81} \times 243$

$243 + 243q + 243q^2 + 243q^3 + 243q^4 = \boxed{363}$

۴-

$\frac{64-1}{6} = \frac{63}{6} = 10.5 \rightarrow a_n = 10.5n - 9.5$

$a_4 = 10.5(4) - 9.5 = 32.5 \rightarrow A$

$q^6 = \frac{64}{1} = q^6 = 64$

$r = \pm 2 \rightarrow a_n = 1 \times \pm 2^{n-1}$

$a_4 = 1 \times 2^{4-1} \rightarrow a_4 = 8 \rightarrow B$

$a_4 = -8 \rightarrow B$

$A + B \rightarrow 32.5 + 8 = \boxed{40.5}$ $\qquad A + B \rightarrow 32.5 - 8 = \boxed{24.5}$

۵-

$d = -\frac{95}{4} + \frac{96}{4} = \frac{1}{4} \rightarrow a_n = \frac{n}{4} - \frac{97}{4} \longrightarrow a_{101} = \frac{101}{4} - \frac{97}{4} = \frac{4}{4} = 1$

a_8 (هندسی) $= 128q^7$ $\qquad 128q^7 = 1$

$q^7 = \frac{1}{128} \rightarrow \boxed{q = \frac{1}{2}}$

۶-

a_3, a_5, a_9

$a_1 + 2d, a + 4d, a + 8d \rightarrow a_1^2 + 16d^2 + 8ad = a^2 + 16d^2 + 10ad \rightarrow 20d^2 = -2ad$

$(a+4d)^2 = (a+8d)(a+2d)$

$2d(10d + a_1) = 0$

جملات یکسان باشند! $\leftarrow \boxed{0}$

$\boxed{d = \frac{-a}{10}}$