-۷

$a_2 ,\quad a_2+2d ,\quad a_2+4d$

$a_8\ a_8+2a \qquad a+3a=4a$

$q_{\text{هندسی}} = 2 \qquad a_{1\,\text{هندسی}} = \frac{1}{4}$

$a_{n\,\text{هندسی}} = \frac{1}{4} \times 2^{n-1}$

$a_{10} = \frac{1}{4} \times 2^9 = \boxed{128}$

$(a_2+2d)^2 = (a)(a+4d) \to \cancel{a^2} + 4d^2 + 4ad = \cancel{a^2} + 4ad$

$4d^2 = 2ad$

$2d = a$

۲ ✓

-۸

$3a_2 , 2a_3 , a_4$

$3(aq) , 2(aq^2) , aq^3 \to 2aq^2 = \frac{3aq + aq^3}{2} \to -4aq^2 + 3aq + aq^3 = 0 \to \underbrace{aq}_{\substack{aq=0 \\ a=0}}(-4q+3+q^2)$

$(q-1)(q-3) = 0$

قابل پذیرش نیست

$q=1$ غق

$\boxed{q=3}$

۲ ✓

-۹

$d = \frac{7}{4} - 2 = -\frac{1}{4} \to \begin{cases} a_4 = 2 + \frac{3}{4} = \frac{5}{4} \\ a_8 = 2 - \frac{7}{4} = \frac{1}{4} \\ a_{13} = 2 - \frac{12}{4} = -1 \end{cases}$

$\frac{5}{4} + x ,\ \frac{1}{4} + x ,\ -1 + x$

$\left(\frac{1}{4}+x\right)^2 = \left(x + \frac{5}{4}\right)(x-1)$

$\frac{1}{16} + x^2 + \frac{1}{2}x = x^2 + \frac{1}{4}x - \frac{5}{4} \to x = -$

$-4 , -5 , -\frac{25}{4} \to q = \frac{\frac{-25}{4}}{\frac{-5}{1}} = \boxed{\frac{5}{4}}$

۲ ✓

-۱

$a + aq^3 + aq^4 = 73$

$t_1 = a_1 = a ,\ t_2 = a_4 = aq^3 ,\ t_{10} = a_3 = aq^4$

$\left.\begin{aligned} t_2 &= aq^3 \\ t_2 &= t_1 + d \end{aligned}\right\} \ aq^3 = a + d \to d = a(q^3 - 1)$

۲ ✓

$t_{10} = aq^4 = a + 9a(q^3-1) = a + 9aq^3 - 9a \to aq^4 - 9aq^3 + 8a = 0 \to$

$a(q^4 - 9q^3 + 8) = 0 ,\ q^3 = \frac{9 \pm \sqrt{81-32}}{2} \begin{cases} (1)\ q^3 = 8 \to q = 2 \\ (2)\ q^3 = 1 \to q = 1 \end{cases}$

$q = 1 \to a + a + a = 73$

$\to a = \frac{73}{3} \to d = a(q^3 - 1) = \frac{73}{3} \times (1-1) = \boxed{0}$

$q = 2$

$\to a + 8a + 64a = 73 \to a = 1 \to d = a(q^3 - 1) = 1 \times (8-1) = 7$

۱- امیدی / روش

۲۰

$a_1 + a_2 + a_3 = 21$

$a_1 \times a_2 \times a_3 = 64 \to \frac{a}{q} + a + aq = 64 \to a^3 = 64$
$a = 4$

$\frac{4}{q} + 4 + 4q = 21$

$\frac{4}{q} + 4q = 17 \to \frac{4q^2+4}{q} = 17 \to 17q = 4 + 4q^2$

$4q^2 - 17q + 4 = 0 \to \Delta = 289 - 64 = 225 \to q$ غیرقابل $\to \frac{17-15}{8} = \boxed{\frac{1}{4}}$, $\to \frac{17+15}{8} = 4$

۲

۲-

$(2x)^2 = (x^2-2)(x^2+4) \to 8, 4, 2$ ($\times\frac{1}{2}$, $\times\frac{1}{2}$) $\quad q = \frac{1}{2}$

$4x^2 = x^4 + 2x^2 - 8$

$2x^2 = x^4 - 8 \to x^4 - 2x^2 - 8 = 0 \to (x^2-4)(x^2+2) = 0$

$x^2 = 4$

$x = \pm 2$

۲

$S_7 = \left(\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}\right) 8 = \left(1 - \frac{1}{128}\right) \times 16 = \frac{127}{128} \times 16 = \frac{127}{8} = \boxed{15\frac{7}{8}}$

۳-

$1 + q + q^2 + q^3 + q^4 = \frac{121}{81} \longrightarrow$ بدیهی است که اگر در جمله اول ضرب شود حاصل ۵ جمله اول را به ما می‌دهد

$243(1 + q + q^2 + q^3 + q^4) = \frac{121}{81} \times 243$

$243 + 243q + 243q^2 + 243q^3 + 243q^4 = \boxed{363}$

۲

۴-

$\frac{64-1}{6} = \frac{63}{6} = 10.5 \to a_n = 10.5n - 9.5$

$a_4 = 10.5(4) - 9.5 = 32.5 \to A$

$q^6 = \frac{64}{1} = q^6 = 64$

$q = \pm 2 \to a_n = 1 \times \pm 2^{n-1}$

$a_4 = 1 \times 2^{4-1} \to a_4 = 8 \to B$

$a_4 = -8 \to B$

۲

$A + B \to 32.5 + 8 = \boxed{40.5}$ $\qquad A + B \to 32.5 - 8 = \boxed{24.5}$

۵-

$d = -\frac{95}{4} + \frac{96}{4} = \frac{1}{4} \to a_n = \frac{n}{4} - \frac{97}{4} \to a_{101} = \frac{101}{4} - \frac{97}{4} = \frac{4}{4} = 1$

a_8 هندسی $= 128q^7 \qquad 128q^7 = 1$

$q^7 = \frac{1}{128} \to \boxed{q = \frac{1}{2}}$

۲

۶-

a_3, a_7, a_9

$a_1 + 2d, a + 6d, a + 8d \to a_1^2 + 36d^2 + 12ad = a^2 + 16d^2 + 10ad \to 20d^2 = -2ad$

$(a+6d)^2 = (a+8d)(a+2d)$

$2d(10d + a_1) = 0$

$\boxed{0}$ — جملات یکسان باشند!

$\boxed{d = \frac{-a}{10}}$

۲