نام و نام خانوادگی ...پریا تیلی...... پاسخنامه تشریحی تکلیف شماره ...۱۲... کلاس ...دهم ریاضی ۱...

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$a_n = \frac{1}{2}, 1, 2, \dots \quad a_n = \frac{1}{2}\times 2^{n-1} = 2^{n-2}$

الف) $a_{10} = \frac{1}{2}\times 2^{9} = 256$

ب) $\frac{a_{17}}{a_{13}} = q^4 = 2^4 = 16$

ج) $a_4 = \frac{1}{2}\times 2^{3} = 4 \quad a_{10} = \frac{1}{2}\times 2^{9} = 256 \quad \sqrt{a_4\times a_{10}} = \sqrt{1024} = 32$

د) $128 = \frac{1}{2}\times 2^{n-1} \Rightarrow 2^{n-1} = 256 \Rightarrow n = 9$

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$\frac{t_8}{t_5} = q^3 = \frac{96}{12} = 8 \Rightarrow q = 2 \quad t_{10} = t_8\times q^2 = 96\times 4 = 384$

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الف) $\overbrace{a_1\times \underbrace{a_2\times a_3\times a_4}_{a_3\times a_3}\times a_5}^{a_3\times a_3} = 243 \Rightarrow (a_3)^5 \Rightarrow a_3 = 3$

ب) $a_1\times a_5 = a_3\times a_3 = 3\times 3 = 9$

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$2^a, 4\sqrt{2}, 2^b \Rightarrow (4\sqrt{2})^2 = 2^a\times 2^b \Rightarrow 32 = 2^{a+b} \Rightarrow a+b = 5$

واسطه حسابی: $\frac{a+b}{2} = \frac{5}{2} = 2/5$

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$a_3 = 1-x$
$a_7 = x$
$a_{11} = x+1$

$a_{11}\times a_3 = a_7\times a_7 \Rightarrow (1-x)(x+1) = x^2 \Rightarrow 1-x^2 = x^2$

$\Rightarrow 1 = 2x^2 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{\sqrt{2}}{2}$

~~$a_1+a_2q+a_3q^2=12$~~ ~~$a_n=a_1q^{n-1}$~~ $a_1+a_4=18$ $a_2+a_3=12$

~~$a_1+a_2+a_3=28$~~ ~~$\Rightarrow a_2^3=27$~~ ~~$\Rightarrow d_3$~~

$\Rightarrow \frac{a(1+q^3)}{aq(1+q)}=\frac{18}{12}=\frac{(1+q)(q^2-q+1)}{q(1+q)}=\frac{7}{4}\Rightarrow 7q=3q^2-3q+3$

$\Rightarrow 3q^2-10q+3=0\Rightarrow q=\frac{10\pm\sqrt{100-36}}{6}$ بزرگترین جمله در هر صورت $\Rightarrow$

$\Rightarrow 3$ $\Rightarrow a_1=1$ $\Rightarrow a_n=1,3,9,27,\dots$

$\Rightarrow \frac{1}{3}$ $\Rightarrow a_1=27$ $\Rightarrow a_n=27,9,3,1,\dots$

$a_1+a_2+a_3=13$ (27) $\Rightarrow a_1+3a_2=13\Rightarrow a_1+a_3=10\Rightarrow a_2q^{-1}+a_2q=10$

$a_1a_2a_3=27\Rightarrow a_2^3=27\Rightarrow a_2=3\Rightarrow a_1a_3=9=a_2q^{-1}\times a_2q$

$3q^{-1}+3q=10\Rightarrow 3+3q^2=10q\Rightarrow 3q^2-10q+3=0\Rightarrow q=\frac{10\pm\sqrt{100-36}}{6}$

$\Rightarrow q=3$

$\Rightarrow q=\frac{1}{3}$

if $q=3\Rightarrow a_n=1,3,9$

if $q=\frac{1}{3}\Rightarrow a_n=9,3,1$

$a_n=\underbrace{2,4,8}_{\times 2\ \times 2},\dots\Rightarrow q=2\Rightarrow a_n=2\times 2^{n-1}$

$S_{10}=2\times\left(\frac{2^{10}-1}{2-1}\right)=2^{10}-1=1023$

الف)

$P_{10}=\sqrt{(a_1a_{10})^{10}}=(a_1\times a_{10})^5=(2\times 2^9)^5=2^{10\times 5}$ ✓

$a_n=a,aq,aq^2,aq^3,\dots\Rightarrow t_n=aq-a,aq^2-aq,aq^3-aq^2,\dots$

صعودی است؟

$\Rightarrow a(q-1),a(q^2-q),a(q^3-q^2),\dots$

$a_2^2=a_1\times a_3\Rightarrow (a(q^2-q))^2=(a(q-1))(a(q^3-q^2))$: اثبات صعودی بودن

$\Rightarrow a^2q^4-2a^2q^3+a^2q^2=a^2q^4-2a^2q^3+a^2q^2$ ✓ برقرار است

پس دنباله t_n صعودی است

الف) $a_n=a_1,a+d,a+2d,a+3d,\dots,a+(n-1)d$

$S_n=\frac{n}{2}(2a+(n-1)d)$

$S_n=a+a+d+a+2d+a+3d+\dots+a+(n-1)d=na+(d+2d+\dots)$

$=na+(1+2+3+\dots+(n-1))d$ حسابی $=na+\frac{(n-1+1)(n-1)}{2}d=na+\frac{n^2-n}{2}d=\frac{n}{2}(2a+(n-1)d)$

$a_n=a,aq,aq^2,\dots,aq^{n-1}$ $S_n=a+aq+aq^2+\dots+aq^{n-1}$ $\times q$ $\Rightarrow S_n\cdot q=aq+aq^2+\dots+aq^n$

$\Rightarrow S_n-S_n\cdot q=a-aq^n\Rightarrow S_n(1-q)=a(1-q^n)\Rightarrow S_n=a\left(\frac{1-q^n}{1-q}\right)=a\left(\frac{q^n-1}{q-1}\right)$