۱۸/۵

نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۱۲ ..... کلاس ..... B .....

۱

الف) $t_n = t_1 \times q^{n-1}$ $\quad t_n = 1 \times 2^{n-1} \Rightarrow \frac{1}{2} \times 2^{n-1} = 2^{8} = 256$ ✓

ب) $a_{12} = \frac{1}{2} \times 2^{11} = 2^{10} \Rightarrow a_1 = \frac{1}{2} \times 2^{1} = 1 \Rightarrow \frac{2^{10}}{2^{11}} = 2^{-1} = \frac{1}{2}$ ✓

ج) $a_4 = \frac{1}{2} \times 2^{3} = 4 \quad a_1 = \frac{1}{2} \times 2^{0} = \frac{1}{2} \quad a_{10} = \frac{1}{2} \times 2^{9} = 2^8 = 256$

$b^2 = ac \Rightarrow b^2 = 4 \times 256 = 1024 \Rightarrow b = \pm 32$ ✓

۱٫۷۵

د) $\frac{1}{2} \times 2^{n-1} = 128 \Rightarrow 2^{n-1} = 2^8 \Rightarrow n = 9$

$2^{n-1} = 256 \rightarrow n-1 = 8 \rightarrow n = 9$

۲

$q^{m-n} = \frac{a_m}{a_n} \Rightarrow q^{12} = \frac{96}{12} = 8 \Rightarrow q = 2$

$t_n = t_1 \times q^{n-1} \Rightarrow t_5 = t_1 \times 2^4 \Rightarrow 12 = t_1 \times 16 \Rightarrow t_1 = \frac{3}{4}$

۲

$a_9 = \frac{3}{4} \times 2^8 = 3 \times 256 = 768$ ✓

۳

الف) $a_3 \times a_4 \times a_5 \times a_6 \times a_7 = 243 \Rightarrow a_5^5 = 243 \Rightarrow a_5 = 3$ ✓

۲

ب) $a_1 \times a_9 = a_5^2 = 3^2 = 9$ ✓

۴

$(4\sqrt{2})^{a+b} = 2^{5a+5b} \Rightarrow 32 = 2^{5a+5b} \Rightarrow 2^5 = 2^{5a+5b} \Rightarrow a+b = 1$

$\frac{a+b}{2} = \frac{1}{2}$ ✓

۲

۵

$x^2 = (x+1)(-x) \Rightarrow x^2 = 1 - x^2 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

۱٫۷۵

فقط حالت ⊕ قابل قبول است زیرا به ازای $x = -\frac{\sqrt{2}}{2}$ جملات فرد با هم غیر هم علامت می شوند

۶

$a + ar^3 = 28 \Rightarrow a(1+r^3) = 28 \Rightarrow a = \frac{28}{1+r^3}$

$ar + ar^2 = 12 \Rightarrow ar(1+r) = 12 \Rightarrow a = \frac{12}{r(1+r)}$ $\Rightarrow \frac{12}{r(1+r)}(1+r^3) = 28$

$\frac{12(1+r^3)}{r(1+r)} = 28 \Rightarrow 12(1+r^3) = 28r(1+r) \Rightarrow 12 + 12r^3 = 28r + 28r^2$

$12r^3 - 28r^2 - 28r + 12 = 0 \Rightarrow 3r^3 - 7r^2 - 7r + 3 = 0 \Rightarrow r = 3 \Rightarrow 3(27)$

$-7(9) - 7(3) + 3 = 0 \quad a = \frac{12}{3+9} = \frac{12}{12} = 1 \Rightarrow a_1, a_2, a_3, a_4 = 1, 3, 9, 27$

(۲)

۷

$P_3 = 27 \Rightarrow t_2 = 3 = aq = 3 \Rightarrow a = \frac{3}{q}$

$S_3 = a + aq + aq^2 = \frac{3}{q}(1 + q + q^2) = 13 \Rightarrow 13q = 3(1+q+q^2) \Rightarrow q = \frac{10 \pm \sqrt{64}}{6}$

$= \frac{10 \pm 8}{6} \Rightarrow q = 3, \frac{1}{3}$

(۲)

$a = \frac{3}{q} \Rightarrow$ 1: $q = 3 \Rightarrow a = 1 \Rightarrow 1, 3, 9$ 2: $q = \frac{1}{3} \Rightarrow a = 9 \Rightarrow 9, 3, 1$

۸

الف)

$a_n = 2, 6, 18, \ldots \quad a_n = 2 \times 3^{n-1}$

$S_n = \frac{2 \times (3^{10} - 1)}{3 - 1} = 3^{10} - 1$

(۲)

$P_n = \sqrt{(2 \times 2 \times 3^9)^{10}} \Rightarrow \sqrt{2^{20} \times 3^{90}} = 2^{10} \times 3^{45}$

$a_{10} = 2 \times 3^9$

۹

$a_n = a, aq, aq^2, aq^3, \ldots$

$b_1 = aq - a = a(q-1)$

$b_2 = aq^2 - aq = aq(q-1)$ $\Rightarrow a(q-1), aq(q-1), aq^2(q-1), \ldots$

$b_3 = aq^3 - aq^2 = aq^2(q-1)$

(۲)

$\frac{b_2}{b_1} = \frac{aq(q-1)}{a(q-1)} = q = r_{new}$

۱۰

ب)

$S_n = a_1 + a_1q + a_1q^2 + \ldots + a_1q^{n-1} \Rightarrow S_nq = a_1q + a_1q^2 + \ldots + a_1q^n$

$S_n - S_nq = a_1 - a_1q^n \Rightarrow S_n(1-q) = a_1(1-q^n) \Rightarrow S_n = \frac{a_1(q^n-1)}{q-1} = \frac{a_1(1-q^n)}{1-q}$

(۱)

الف)

$2S = (2a + (n-1)d) + (2a + (n-1)d) + \ldots + (2a + (n-1)d) \Rightarrow 2S = n \times (2a + (n-1)d)$

$\Rightarrow S = \frac{n}{2} \times (2a + (n-1)d)$

$S_n = a_1 + a_2 + a_3 + \ldots + a_n \longrightarrow S_n = a_1 + a_1 + d + a_1 + 2d + \ldots + a_1 + (n-1)d$

$S_n = na_1 + (1 + 2 + \ldots + (n-1))d \rightarrow S_n = na_1 + \frac{n(n-1)}{2}d$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$